

MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Polar Coordinates - F2 (Pearson Edexcel)

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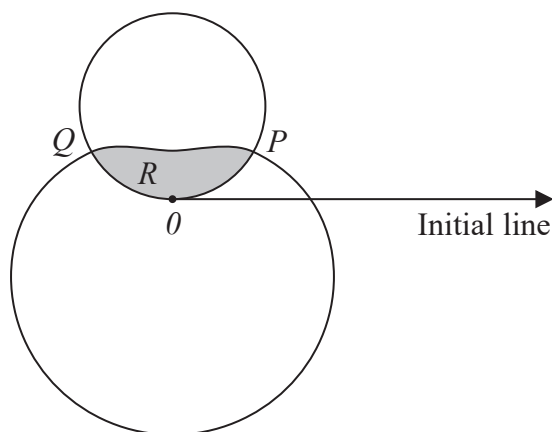


Figure 1

Figure 1 shows a sketch of the curves with polar equations

$$r = 2 \sin \theta \qquad 0 \leq \theta \leq \pi$$

$$r = 1.5 - \sin \theta \quad 0 \leq \theta \leq 2\pi$$

The curves intersect at the points P and Q .

- (a) Find the polar coordinates of the point P and the polar coordinates of the point Q . (3)

The region R , shown shaded in Figure 1, is enclosed by the two curves.

- (b) Find the exact area of R , giving your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational numbers to be found.
- (8)**



Question Number	Scheme	Notes	Marks
8(a)	$2\sin\theta = 1.5 - \sin\theta \Rightarrow \theta = \dots$ or $\sin\theta = \frac{r}{2} \Rightarrow r = 1.5 - r \Rightarrow r = \dots$	Equate and attempt to solve for θ or Eliminates $\sin\theta$ and solves for r	M1
	$P\left(1, \frac{\pi}{6}\right)$	Correct coordinates. Allow the marks as soon as the correct values are seen and allow coordinates the wrong way round and allow awrt 0.524 for $\pi/6$	A1
	$Q\left(1, \frac{5\pi}{6}\right)$	Correct coordinates. Allow the marks as soon as the correct values are seen and allow coordinates the wrong way round and allow awrt 2.62 for $5\pi/6$	A1
			(3)

(b)	$\left(\frac{1}{2}\right) \int (1.5 - \sin \theta)^2 d\theta \text{ or } \left(\frac{1}{2}\right) \int (2 \sin \theta)^2 d\theta$ <p>Attempts to use $\dots \int (\sin \theta)^2 d\theta$ or $\dots \int (1.5 - \sin \theta)^2 d\theta$</p>	M1
	$(1.5 - \sin \theta)^2 = 2.25 - 3 \sin \theta + \sin^2 \theta = 2.25 - 3 \sin \theta + \frac{(1 - \cos 2\theta)}{2}$ <p>Expands (allow poor squaring e.g. $(1.5 - \sin \theta)^2 = 2.25 + \sin^2 \theta$ and attempts to use</p> $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{\cos 2\theta}{2}$	M1
	$\frac{1}{2} \int (1.5 - \sin \theta)^2 d\theta = \frac{1}{2} \left[\frac{11}{4} \theta + 3 \cos \theta - \frac{1}{4} \sin 2\theta \right]$ <p>M1: Attempt to integrate and reaches an expression of the form $\alpha \theta + \beta \cos \theta + \gamma \sin 2\theta$</p> <p>A1: Correct integration (with or without the $\frac{1}{2}$)</p>	M1A1
	$\frac{1}{2} \left[\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \frac{1}{2} \left\{ \left(\frac{11}{4} \cdot \frac{5\pi}{6} + 3 \cdot \cos \frac{5\pi}{6} - \frac{1}{4} \sin 2 \cdot \frac{5\pi}{6} \right) - \left(\frac{11}{4} \cdot \frac{\pi}{6} + 3 \cdot \cos \frac{\pi}{6} - \frac{1}{4} \sin 2 \cdot \frac{\pi}{6} \right) \right\}$ <p>This is a key step and must be the correct method for this part of the area e.g. uses their $\frac{\pi}{6}$ and their $\frac{5\pi}{6}$ (or twice limits of their $\frac{\pi}{6}$ and $\frac{\pi}{2}$)</p>	M1
	$\frac{1}{2} \int (2 \sin \theta)^2 d\theta = \int (1 - \cos 2\theta) d\theta = \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{6}} = \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) - (-0)$ <p>Uses the limits 0 and their $\frac{\pi}{6}$ to find at least one segment.</p> <p>If using integration, must have integrated to obtain $p\theta + q \sin 2\theta$ with correct use of limits</p> <p>NB can be done as: $\frac{1}{2} (1)^2 \left(\frac{\pi}{3} \right) - \frac{1}{2} (1)^2 \sin \left(\frac{\pi}{3} \right)$ but must be correct work for their angles</p>	M1
	$\frac{11}{12} \pi - \frac{11\sqrt{3}}{8} + 2 \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) = \frac{5}{4} \pi - \frac{15}{8} \sqrt{3}$ <p>ddM1: Adds their two areas to give a numerical value for the shaded area Dependent on the previous 2 M marks and must be a completely correct strategy so needs to be an attempt at:</p> $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1.5 - \sin \theta)^2 d\theta \text{ or } 2 \times \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1.5 - \sin \theta)^2 d\theta$ <p style="text-align: center;">+</p> $2 \times \frac{1}{2} \int_0^{\frac{\pi}{6}} (2 \sin \theta)^2 d\theta \text{ or } \left(\frac{1}{2} \int_0^{\frac{\pi}{6}} (2 \sin \theta)^2 d\theta + \frac{1}{2} \int_{\frac{5\pi}{6}}^{\pi} (2 \sin \theta)^2 d\theta \right)$ <p>A1: Correct answer (allow equivalent fractions)</p>	ddM1A1
		(8)
		Total 11

	<p>Note that attempts to use $\left(\frac{1}{2}\right) \int (C_1 - C_2)^2 d\theta$ e.g. $\left(\frac{1}{2}\right) \int (2 \sin \theta - (1.5 - \sin \theta))^2 d\theta$</p> <p>Will probably only score a maximum of the first 3 marks i.e.</p> <p>M1 for $\left(\frac{1}{2}\right) \int (2 \sin \theta - (1.5 - \sin \theta))^2 d\theta$</p> <p>M1 for expanding and attempting to use $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{\cos 2\theta}{2}$</p> <p>M1 for attempting to integrate and reaching an expression of the form $\alpha\theta + \beta \cos \theta + \gamma \sin 2\theta$</p>	M1
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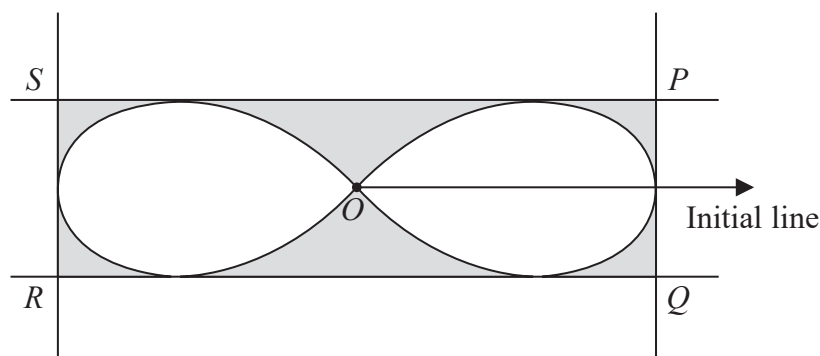


Figure 1

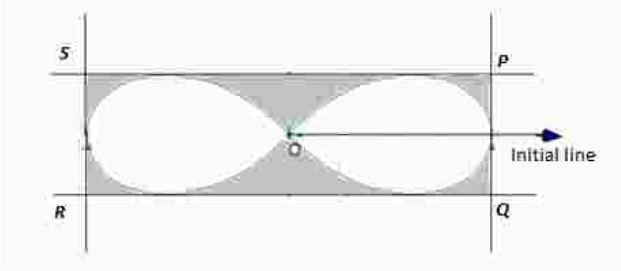
Figure 1 shows a sketch of the curve C with polar equation

$$r = 4 \cos 2\theta, \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \quad \text{and} \quad \frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4}$$

The lines PQ , QR , RS and SP are tangents to C , where QR and SP are parallel to the initial line and PQ and RS are perpendicular to the initial line.

- (a) Find the polar coordinates of the points where the tangent SP touches the curve. Give the values of θ to 3 significant figures. (5)
- (b) Find the exact area of the finite region bounded by the curve C , shown unshaded in Figure 1. (5)
- (c) Find the area enclosed by the rectangle $PQRS$ but outside the curve C , shown shaded in Figure 1. (5)



Question Number	Scheme	Notes	Marks
7			
(a)	$y = r \sin \theta = 4 \cos 2\theta \sin \theta$	Attempts to use $r \sin \theta$	M1
	$\frac{dy}{d\theta} = 4 \cos 2\theta \cos \theta - 8 \sin 2\theta \sin \theta$ <p>or</p> $y = 4(1 - 2 \sin^2 \theta) \sin \theta = 4 \sin \theta - 8 \sin^3 \theta \Rightarrow \frac{dy}{d\theta} = 4 \cos \theta - 24 \sin^2 \theta \cos \theta$ <p>A correct expression for $\frac{dy}{d\theta}$ or any multiple of $\frac{dy}{d\theta}$</p>		B1
	$\frac{dy}{d\theta} = 0 \Rightarrow \theta = \dots$	Set their $\frac{dy}{d\theta} = 0$ and attempt to solve to obtain a value for θ	M1
	$r = \frac{8}{3}, \theta = 0.421, \theta = 2.72$	Any one of: $r = \frac{8}{3}$ (or awrt 2.7) or $\theta = 0.421 \dots$ or $\theta = 2.72 \dots$	A1
	$r = \frac{8}{3}$ $\theta = 0.421, 2.72$	Correct value for r and both angles correct. May be seen as $\left(\frac{8}{3}, 0.421\right), \left(\frac{8}{3}, 2.72\right)$. Allow $\left(0.421, \frac{8}{3}\right), \left(2.72, \frac{8}{3}\right)$ but coordinates do not have to be paired and accept awrt 0.421, 2.72 and allow awrt 2.7 for $\frac{8}{3}$. Ignore any other coordinates given once the correct values have been seen.	A1
			(5)

(b)	$A = \dots \int (4 \cos 2\theta)^2 d\theta$	Indication that the integration of $(4 \cos 2\theta)^2$ is required. Ignore any limits and ignore any constant factors at this stage.	M1
	$\cos^2 2\theta = \frac{1}{2}(1 + \cos 4\theta)$	A correct identity seen or implied.	A1
	$A = \dots [\alpha\theta + \beta \sin 4\theta]$	Integrates to obtain an expression of the form $\alpha\theta + \beta \sin 4\theta$. Ignore any limits and ignore any constant factors. Dependent on the first method mark.	dm1
	$= 16 \left[\theta + \frac{1}{4} \sin 4\theta \right]_0^{\frac{\pi}{4}}$	A fully correct method that if evaluated correctly would give the answer 4π . Note that the correct “constant factor” may only be applied at the very last stage of their working and this method mark would only be awarded at that point. Dependent on all previous method marks.	ddM1
	Examples that could score the final M1 (following correct work): $16 \left[\theta + \frac{1}{4} \sin 4\theta \right]_0^{\frac{\pi}{4}}, 8 \left[\theta + \frac{1}{4} \sin 4\theta \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}, 8 \left[\theta + \frac{1}{4} \sin 4\theta \right]_0^{\frac{\pi}{2}}, 16 \left[\theta + \frac{1}{4} \sin 4\theta \right]_{\frac{3\pi}{4}}^{\pi}$		
	$= 4\pi$	cao	A1
			(5)

(c)	$PQ = 2r \sin \theta = \frac{16}{3\sqrt{6}}$	Correct expression or value for PQ or $PQ/2$. E.g. $2\left(\frac{8}{3}\right)\frac{1}{\sqrt{6}}$, $2\left(\frac{8}{3}\right)\sin 0.421$, $2\left(\frac{8}{3}\right)\sin 2.72$, $\frac{8\sqrt{6}}{9}$ or half of these. May be implied by awrt 2.2 or awrt 1.1	B1
	$SP = 8$ or $\frac{SP}{2} = 4$	Correct value for SP or $SP/2$	B1
	Area $PQRS = \frac{16}{3\sqrt{6}} \times 8 \left(= \frac{64\sqrt{6}}{9} \right)$	Their $PQ \times SP$. Must be the complete rectangle here.	M1
	Required area = $\frac{128}{3\sqrt{6}} - 4\pi$	M1: Their rectangle area – their answer to part (b)	M1A1
		A1: Correct exact answer or equivalent exact form e.g. $\frac{64\sqrt{6}}{9} - 4\pi$ or allow awrt 4.8 or 4.9	
			(5)
			Total 15

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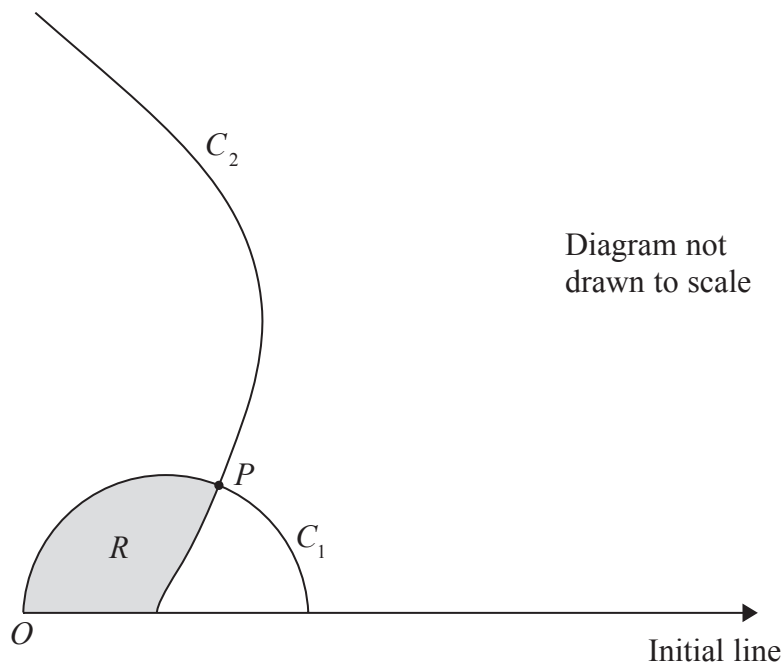


Figure 1

Figure 1 shows a sketch of the curves C_1 and C_2 with polar equations

$$C_1 : r = \frac{3}{2} \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$C_2 : r = 3\sqrt{3} - \frac{9}{2} \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

The curves intersect at the point P .

(a) Find the polar coordinates of P .

(3)

The region R , shown shaded in Figure 1, is enclosed by the curves C_1 and C_2 and the initial line.

(b) Find the exact area of R , giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(8)



Question Number	Scheme	Notes	Marks
7.	$C_1: r = \frac{3}{2} \cos \theta, \quad C_2: r = 3\sqrt{3} - \frac{9}{2} \cos \theta$		
(a)	$\frac{3}{2} \cos \theta = 3\sqrt{3} - \frac{9}{2} \cos \theta \Rightarrow \theta = \dots$ or $\cos \theta = \frac{2r}{3} \Rightarrow r = 3\sqrt{3} - 3r \Rightarrow r = \dots$	Puts $C_1 = C_2$ and attempt to solve for θ or Eliminates $\cos \theta$ and solves for r	M1
	$\theta = \frac{\pi}{6} \quad \text{or} \quad r = \frac{3\sqrt{3}}{4}$	Correct θ or correct r . Allow $\theta = \text{awrt } 0.524, r = \text{awrt } 1.3$	A1
	$r = \frac{3\sqrt{3}}{4} \quad \text{and} \quad \theta = \frac{\pi}{6}$	Correct r and θ (isw e.g. $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{4}\right)$) Allow $\theta = \text{awrt } 0.524, r = \text{awrt } 1.3$	A1
			(3)

7(b)	$\frac{1}{2} \int \left(3\sqrt{3} - \frac{9}{2} \cos \theta \right)^2 d\theta \quad \text{or} \quad \frac{1}{2} \int \left(\frac{3}{2} \cos \theta \right)^2 d\theta$		M1
	Attempts to use correct formula on either curve. The $\frac{1}{2}$ may be implied by later work.		
	$\left(3\sqrt{3} - \frac{9}{2} \cos \theta \right)^2 = 27 - 27\sqrt{3} \cos \theta + \frac{81}{4} \cos^2 \theta = 27 - 27\sqrt{3} \cos \theta + \frac{81}{4} \frac{(\cos 2\theta + 1)}{2}$		M1
	Expands to obtain an expression of the form $a + b \cos \theta + c \cos^2 \theta$ and attempts to use $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{\cos 2\theta}{2}$		
	$\left(\frac{1}{2} \right) \int \left(3\sqrt{3} - \frac{9}{2} \cos \theta \right)^2 d\theta = \left(\frac{1}{2} \right) \left[\frac{297}{8} \theta - 27\sqrt{3} \sin \theta + \frac{81}{16} \sin 2\theta \right]$		M1A1
	M1: Attempts to integrate to obtain at least two terms from $\alpha \theta$, $\beta \sin \theta$, $\gamma \sin 2\theta$ A1: Correct integration with or without the $\frac{1}{2}$ (NB $\frac{297}{8} = 27 + \frac{81}{8}$)		
	$\left(\frac{1}{2} \right) \left[\frac{297}{8} \theta - 27\sqrt{3} \sin \theta + \frac{81}{16} \sin 2\theta \right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \left(\frac{1}{2} \right) \left\{ \left(\frac{297}{8} \cdot \frac{\pi}{6} - 27\sqrt{3} \cdot \sin \frac{\pi}{6} + \frac{81}{16} \sin 2 \cdot \frac{\pi}{6} \right) - (-0) \right\}$		M1
	M1: Uses the limits 0 and their $\frac{\pi}{6}$ If the substitution for $\theta = 0$ evaluates to 0 then the substitution for $\theta = 0$ does not need to be seen but if it does not evaluate to 0, the substitution for $\theta = 0$ needs to be seen.		
	$\frac{1}{2} \int \left(\frac{3}{2} \cos \theta \right)^2 d\theta = \frac{9}{16} \int (\cos 2\theta + 1) d\theta = \frac{9}{16} \left[\frac{1}{2} \sin 2\theta + \theta \right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \frac{9}{16} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right)$		M1
	M1: Uses $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{\cos 2\theta}{2}$, integrates to obtain at least $k \sin 2\theta$ and uses the limits of their $\frac{\pi}{6}$ and $\frac{\pi}{2}$ to find the other area NB can be done as a segment : $\frac{1}{2} \left(\frac{3}{4} \right)^2 \left(\frac{2\pi}{3} \right) - \frac{1}{2} \left(\frac{3}{4} \right)^2 \sin \left(\frac{\pi}{3} \right)$ Allow $\frac{1}{2} \left(\frac{3}{4} \right)^2 \left(\pi - 2 \times \text{their } \frac{\pi}{6} \right) - \frac{1}{2} \left(\frac{3}{4} \right)^2 \sin \left(\pi - 2 \times \text{their } \frac{\pi}{6} \right)$		
	$\frac{297}{96} \pi - \frac{351\sqrt{3}}{64} + \frac{9}{16} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) = \frac{105}{32} \pi - \frac{45}{8} \sqrt{3}$		M1A1
	M1: Adds their two areas both of which are of the form $a\pi + b\sqrt{3}$ A1: Correct answer (allow equivalent fractions for $\frac{105}{32}$ and/or $\frac{45}{8}$)		
			(8)
			Total 11

Special Case – Uses $\pm(C_1 - C_2)$

(b)	$\frac{1}{2} \int \left(3\sqrt{3} - \frac{9}{2} \cos \theta - \frac{3}{2} \cos \theta \right)^2 d\theta$	M1
	Attempts to use correct formula on $\pm(C_1 - C_2)$. The $\frac{1}{2}$ may be implied by later work.	
	$\left(3\sqrt{3} - 6 \cos \theta \right)^2 = 27 - 36\sqrt{3} \cos \theta + 36 \cos^2 \theta = 27 - 36\sqrt{3} \cos \theta + 36 \frac{(\cos 2\theta + 1)}{2}$	M1
	Expands to obtain an expression of the form $a + b \cos \theta + c \cos^2 \theta$ and attempts to use $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{\cos 2\theta}{2}$	
	$\left(\frac{1}{2} \right) \int \left(3\sqrt{3} - 6 \cos \theta \right)^2 d\theta = \left(\frac{1}{2} \right) [45\theta - 36\sqrt{3} \sin \theta + 9 \sin 2\theta]$	M1
	Attempts to integrate to obtain at least two terms from $\alpha\theta$, $\beta \sin \theta$, $\gamma \sin 2\theta$	
	No more marks available	

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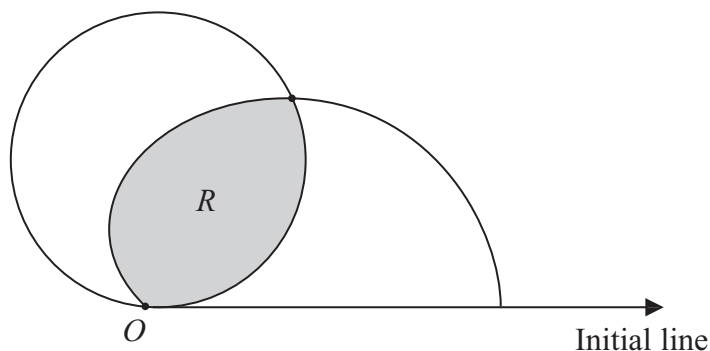


Figure 1

$$r = \sqrt{3} \sin \theta, \quad 0 \leq \theta \leq \pi$$

$$r = 1 + \cos \theta, \quad 0 \leq \theta \leq \pi$$

- (a) Verify that the curves intersect at the point P with polar coordinates $\left(\frac{3}{2}, \frac{\pi}{3}\right)$. **(2)**

The region R , bounded by the two curves, is shown shaded in Figure 1.

- (b) Use calculus to find the exact area of R , giving your answer in the form $a(\pi - \sqrt{3})$, where a is a constant to be found.
- (6)**



Question Number	Scheme	Notes	Marks
7(a)	$\theta = \frac{\pi}{3} \Rightarrow r = \sqrt{3} \sin\left(\frac{\pi}{3}\right) = \frac{3}{2}$	Attempt to verify coordinates in at least one of the polar equations	M1
	$\theta = \frac{\pi}{3} \Rightarrow r = 1 + \cos\left(\frac{\pi}{3}\right) = \frac{3}{2}$	Coordinates verified in both curves (Coordinate brackets not needed)	A1
			(2)
	Alternative:		
	Equate rs : $\sqrt{3} \sin \theta = 1 + \cos \theta$ and verify (by substitution) that $\theta = \frac{\pi}{3}$ is a solution or solve by using $t = \tan \frac{\theta}{2}$ or writing $\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta = \frac{1}{2}$ $\sin\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2}$ $\theta = \frac{\pi}{3}$ Squaring the original equation allowed as θ is known to be between 0 and π		M1
	Use $\theta = \frac{\pi}{3}$ in either equation to obtain $r = \frac{3}{2}$		A1
(b)	$\frac{1}{2} \int (\sqrt{3} \sin \theta)^2 d\theta, \quad \frac{1}{2} \int (1 + \cos \theta)^2 d\theta$	Correct formula used on at least one curve (1/2 may appear later) Integrals may be separate or added or subtracted.	M1
	$= \frac{1}{2} \int 3 \sin^2 \theta d\theta, \quad \frac{1}{2} \int (1 + 2 \cos \theta + \cos^2 \theta) d\theta$		
	$= \left(\frac{1}{2}\right) \int \frac{3}{2} (1 - \cos 2\theta) d\theta, \quad \left(\frac{1}{2}\right) \int (1 + 2 \cos \theta + \frac{1}{2} (1 + \cos 2\theta)) d\theta$ Attempt to use $\sin^2 \theta$ or $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ on either integral Not dependent 1/2 may be missing		M1
	$= \frac{3}{4} \left[\theta - \frac{1}{2} \sin 2\theta \right]_{(0)}^{\left(\frac{\pi}{3}\right)}, \quad \frac{1}{2} \left[\frac{3}{2} \theta + 2 \sin \theta + \frac{1}{4} \sin 2\theta \right]_{\left(\frac{\pi}{3}\right)}^{(\pi)}$ Correct integration (ignore limits) A1A1 or A1A0		A1, A1
	$R = \frac{3}{4} \left[\frac{\pi}{3} - \frac{\sqrt{3}}{4} (-0) \right] + \frac{1}{2} \left[\frac{3\pi}{2} - \left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} \right) \right]$	Correct use of limits for both integrals Integrals must be added. Dep on both previous M marks	ddM1
	$= \frac{3}{4} (\pi - \sqrt{3})$	Cao No equivalents allowed	A1
			(6)
			Total 8

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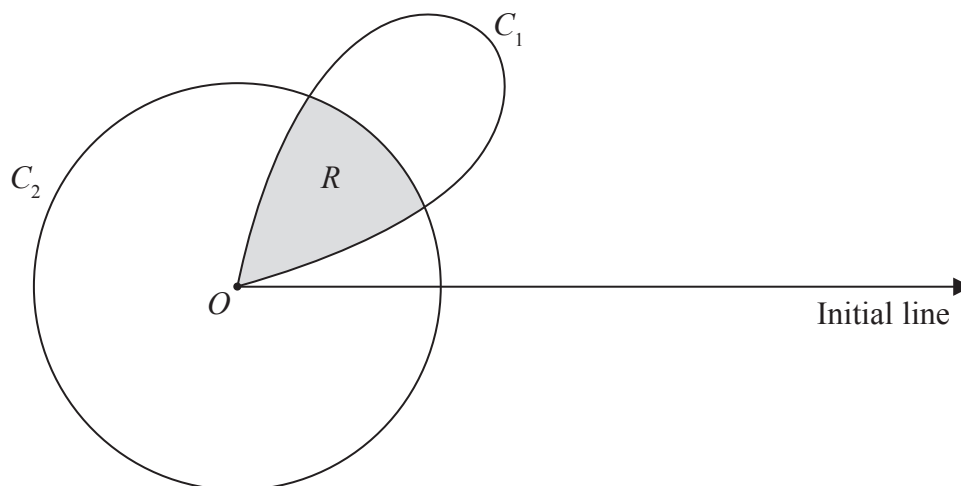


Figure 1

Figure 1 shows the curve C_1 with polar equation $r=2a \sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and the circle C_2 with polar equation $r=a$, $0 \leq \theta \leq 2\pi$, where a is a positive constant.

- (a) Find, in terms of a , the polar coordinates of the points where the curve C_1 meets the circle C_2
- (3)**

The regions enclosed by the curve C_1 and the circle C_2 overlap and the common region R is shaded in Figure 1.

- (b) Find the area of the shaded region R , giving your answer in the form $\frac{1}{12}a^2(p\pi + q\sqrt{3})$, where p and q are integers to be found.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question Number	Scheme		Marks
9.			
(a)	$a = 2a \sin 2\theta \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \dots$	$C_1 = C_2$ and attempt to solve for 2θ	M1
	$\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	$2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or both Decimals allowed (min 3 sf).	A1
	$\left(a, \frac{\pi}{12}\right), \left(a, \frac{5\pi}{12}\right)$	Both points Can be written $r = a, \theta = \frac{\pi}{12}, \frac{5\pi}{12}$ Decimals allowed (min 3 sf).	A1
			(3)
(b)	$\frac{1}{2} \times a^2 \times \frac{\pi}{3}$ oe	Correct expression for the sector	B1
	$\frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (2a \sin 2\theta)^2 d\theta$	Use of correct formula Limits not needed (ignore any shown)	M1
	$\cos 4\theta = 1 - 2\sin^2 2\theta$ $\Rightarrow \sin^2 2\theta = \frac{1}{2}(1 - \cos 4\theta)$	Uses $\sin^2 2\theta = \frac{\pm 1 \pm \cos 4\theta}{2}$	M1
	$\int (1 - \cos 4\theta) d\theta = \theta - \frac{1}{4} \sin 4\theta$	Correct integration Limits not needed (ignore any shown)	A1
	$I = a^2 \left[\theta - \frac{1}{4} \sin 4\theta \right]_0^{\frac{\pi}{12}}$ $= a^2 \left\{ \left(\frac{\pi}{12} - \frac{1}{4} \sin 4 \cdot \frac{\pi}{12} \right) - (0) \right\}$	An attempt to find one or both of the regions either side of the sector. ie uses limits $0, \frac{\pi}{12}$ and/or $\frac{5\pi}{12}, \frac{\pi}{2}$, limits to be substituted and subtracted (if non-zero after substitution). Limits to be used the correct way round. If two integrals seen award mark if either correct. Both previous method marks must have been scored.	ddM1
	$R = 2I + \frac{a^2 \pi}{6} = 2a^2 \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8} \right) + \frac{a^2 \pi}{6}$	Correct strategy for the complete area (sector + $2I$). All areas must be positive.	M1
	$R = \frac{1}{12} a^2 (4\pi - 3\sqrt{3})$	If decimals seen anywhere (either in rt 3 or the limits) this mark is lost.	A1
			(7)
			Total 10