MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Exponentials and Logarithms - C12 (Pearson Edexcel)

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10. (i) Use the laws of logarithms to solve the equation

$$3\log_8 2 + \log_8(7 - x) = 2 + \log_8 x$$

(5)

(ii) Using algebra, find, in terms of logarithms, the exact value of y for which

$$3^{2y} + 3^{y+1} = 10$$

(5)

Question Number	Scheme	Notes	Marks
10(i)	**Examples:** $3\log_8 2 = \log_8 2^3$, $3\log_8 2 = \log_8 8$, $3\log_8 2 = 1$, $\log_8 2 = \frac{1}{3}$, $2 = \log_8 64$	Demonstrates a law or property of logs on either of the constant terms.	B1
	Examples: $\log_8(7-x) - \log_8 x = \log_8 \frac{(7-x)}{x}$, $\log_8 64 + \log_8 x = \log_8 64x$, $\log_8 8 + \log_8(7-x) = \log_8 8(7-x)$	Demonstrates the addition or subtraction law of logs on two terms, at least one of which is in terms of x.	B1
	For the B marks above, look for work as described and award the marks where possible. If there is some correct and some incorrect work, do not look to penalise for the incorrect statements.		
	$\log_8 8(7-x) = \log_8 64x$, $\log_8 \frac{(7-x)}{x} = 1$, $\log_8 \frac{(7-x)}{8x} = 0$, $\log_8 \frac{8(7-x)}{x} = 2$ Correct processing leading to one of these equations or the equivalent. **NB needs to be a correct equation.**		M1
	$8(7-x) = 64x$, $\frac{(7-x)}{x} = 8$, $\frac{7-x}{8x} = 1$, $\frac{8(7-x)}{x} = 64$ **Correct equation with logs removed**		A1
	$x = \frac{7}{9}$	Accept equivalents but must be exact e.g. $\frac{56}{72}$ or 0.777… or 0.7 with a dot over the 7	A1
			(5)
(ii)	$3^{2y} + 3^{y+1} = 10$		
	$3^y \times 3^y + 3\times 3^y = 10$ or $3^y(3^y + 3) = 10$ or $(3^y)^2 + 3\times 3^y = 10$ or $x = 3^y \Rightarrow x^2 + 3x = 10$ A correct quadratic in x (or 3^y)		B1
	$x^2 + 3x - 10 = 0 \Rightarrow x = ...$	Correct attempt to solve a quadratic equation of the form $ax^2 + bx \pm 10 = 0$ (may be a letter other than x or may be 3^y etc.)	M1
	$x = 2$ or $x = 2$ and -5	Correct values.	A1
	$3^y = 2 \Rightarrow y = \log_3 2$ or $\frac{\log 2}{\log 3}$	Correct use of logs. Need to see $3^y = k \Rightarrow y = \log_3 k$ or $\frac{\log k}{\log 3}, k > 0$ which may be implied by awrt 0.63. Allow lg and ln for log.	dM1
	$y = \log_3 2$ or $y = \frac{\log 2}{\log 3}$	Cao (And no incorrect work using "-5"). Give BOD but penalise very sloppy notation e.g. log3(2) for $\log_3 2$ if necessary.	A1
			(5)
			Total 10

(ii) Way 2	$3^{2y}+3^{y+1}=10$		
	$3^{2y}+3^{y+1}=\left(3^2\right)^y+3(9)^{0.5y}$ $\Rightarrow 9^y+3(9)^{0.5y}=10$	Correct quadratic in $9^{0.5y}$	B1
	$x^2+3x-10=0\Rightarrow x=2\left(\text{or } -5\right)$	M1: Correct attempt to solve a quadratic equation of the form $ax^2+bx-10=0$ (may be a letter other than x or may be $9^{0.5y}$ etc.) A1: Correct solution(s)	M1A1
	$9^{0.5y}=2\Rightarrow 0.5y=\log_9 2$ or $\dfrac{\log 2}{\log 9}$	Correct use of logs. Need to see $9^{0.5y}=k\Rightarrow 0.5y=\log_9 k$ or $\dfrac{\log k}{\log 9}, k>0$	dM1
	$y=2\log_9 2$ or $y=\dfrac{2\log 2}{\log 9}$	Cao (And no incorrect work using "-5")	A1
			(5)

Leave blank

6. Find the exact values of x for which

$$2\log_5(x + 5) - \log_5(2x + 2) = 2$$

Give your answers as simplified surds.

(7)

Question Number	Scheme	Marks
6.	Use or state $2\log_5(x+5)=\log_5(x+5)^2$	M1
	Use or states $\log_5(x+5)^2-\log_5(2x+2)=\log_5\frac{(x+5)^2}{(2x+2)}$ or $\log_5(2x+2)+\log_5 5^2=\log_5 5^2(2x+2)$ etc	M1
	Use or state $\log_5 25=2$	M1
	$(x+5)^2=25(2x+2)$ or equivalent	A1
	$x^2-40x-25=0$	A1
	Solves their quadratic to give $x=$ (use formula, calculator or completing the square)	M1
	$x=20\pm5\sqrt{17}$	A1
		[7]
		7 marks

Notes

M1: Uses or states $2\log_5(x+5)=\log_5(x+5)^2$ Can be scored without sight of the base 5 of the log

M1: Uses addition (or subtraction) law correctly at least once. Can be scored without sight of the base 5 on the log

This may follow an incorrect line. Eg. $\log_5 2(x+5)-\log_5(2x+2)=\log_5\frac{2(x+5)}{(2x+2)}$ would be fine for this mark as would $\log_5 10+\log_5(2x+2)=\log_5 10(2x+2)$ but $2\log_5(x+5)-\log_5(2x+2)=2\log_5\frac{(x+5)}{(2x+2)}$ would not score this mark as it is incorrect subtraction law. If the lhs is going to score this mark, the coefficient of "2" must have been dealt with.

M1: Connects 2 with 25 OR 5^2 correctly

A1: Correct equation, not involving logs, in any form (un-simplified). **Dependent upon all 3 M's being awarded**.

A1: Obtains correct 3TQ **Dependent upon all 3 M's being awarded.**

M1: Solves a 3TQ by formula, calculator or completing the square to give a surd answer.

A1: CSO $x=20\pm5\sqrt{17}$

If they reject one of the solutions, usually $x=20-5\sqrt{17}$ then withhold the final mark.

....

There are students who make two or more errors and fortuitously manage to form the correct equation.

Eg $2\log_5(x+5)-\log_5(2x+2)=2\Rightarrow\frac{2\log_5(x+5)}{\log_5(2x+2)}=2\Rightarrow\frac{\log_5(x+5)^2}{\log_5(2x+2)}=2\Rightarrow\frac{(x+5)^2}{(2x+2)}=5^2$

This student scores M1 (shown) M0 (incorrect subtraction law), M1 (shown).

As they have not scored the 3 M marks they only have access to the final M for a total 3 out of 7

Students who start $2\log_5(x+5)=2\log_5 2+2\log_5 5$ will only have access to M3

Leave blank

13. (i) Find the value of x for which

$$4^{3x+2} = 3^{600}$$

giving your answer to 4 significant figures.

(3)

(ii) Given that

$$\log_a (3b - 2) - 2\log_a 5 = 4, \qquad a > 0, a \neq 1, b > \frac{2}{3}$$

find an expression for b in terms of a.

(4)

Question Number	Scheme	Notes	Marks
13(i)	$\log 4^{3x+2} = (3x+2)\log 4\left(\text{allow } 3x+2\log 4\right)$ $\log 3^{600} = 600\log 3$ $\log_4 4^{3x+2} = 3x+2$ $\log_3 3^{600} = 600$ $3x+2 = \log_4 3^{600}$	Evidence of the application of the power law of logarithms or the definition of a logarithm. This is independent of any other working – see examples. Generally this is for e.g. $\log_x y^k = k\log_x y$ or $\log_x x^k = k$ or $\log y^k = k\log y$ etc. where x, y and k are any variables/numbers.	M1
	Examples: $x = \frac{1}{3}\left(\frac{600\log 3}{\log 4} - 2\right)$ **or** $x = \frac{600\log_4 3 - 2}{3}$ **or** $x = \frac{\frac{600}{\log_3 4} - 2}{3}$	This mark is for a correct expression or a correct value for x. Note that it must be an expression that can be evaluated e.g. $x = \frac{\log_4 3^{600} - 2}{3}$ is A0. May be implied by awrt 158 following correct work.	A1
	$x = 157.8$	Cao (Must be this value **not** awrt)	A1
			(3)
(ii)	$2\log_a 5 = \log_a 25$ or $\log_a 5^2$		B1
	$\log_a(3b-2) - \log_a 25 = \log_a \frac{(3b-2)}{25}$ or $\log_a 25 + \log_a a^4 = \log_a 25a^4$	Correct use of subtraction or addition rule	M1
	$a^4 = \frac{3b-2}{25}$	Removes logs correctly. **Dependent on the previous M.**	dM1
	$b = \frac{25a^4+2}{3}$	Cao oe e.g. $b = \frac{25a^4}{3} + \frac{2}{3}$	A1
			(4)
	Special Case: $\log_a(3b-2) - \log_a 25 = \log_a \frac{25}{3b-2} \Rightarrow a^4 = \frac{25}{3b-2}$ Scores B1M0dM1A0		
			Total 7

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5. (a) Given that

$$y = \log_3 x$$

find expressions in terms of y for

(i) $\log_3\left(\dfrac{x}{9}\right)$

(ii) $\log_3 \sqrt{x}$

Write each answer in its simplest form.

(3)

(b) Hence or otherwise solve

$$2\log_3\left(\frac{x}{9}\right) - \log_3 \sqrt{x} = 2$$

(4)

Question Number	Scheme		Marks
5 (a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=y-2$	M1: $\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9$ or $\log_3\left(\frac{x}{9}\right)=\log_3 x+\log_3\frac{1}{9}$ Correct use of the subtraction rule or addition rule. Ignore the presence or absence of a base and any spurious "= 0"	M1A1
		A1: $y-2$	
	An answer left as $\log_3 3^{y-2}$ scores M1A0		
	Note that $\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=y-\log_3 9$ scores M1A0		
(ii)	$\log_3\sqrt{x}=\log_3 x^{\frac{1}{2}}=\frac{1}{2}\log_3 x=\frac{1}{2}y$	$\frac{1}{2}y$ or equivalent	B1
			(3)
(b)	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\Rightarrow 2(y-2)-\frac{1}{2}y=2$ Uses their answers from part (a) to create a **linear** equation in *y* (condone poor use of brackets e.g. $2(y-2)=2y-2$ and also the slip $(y-2)-\frac{1}{2}y=2$ for this mark)		M1
	$\Rightarrow y=4$	Correct value for *y*.	A1
	Note that arriving at $(y-2)^2-\frac{1}{2}y=2$ above scores M0 (not linear) but does have a solution *y* = 4 so look out for *y* = 4 not being derived correctly.		
	$\log_3 x=4\Rightarrow x=3^4$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1
			(4)
			(7 marks)
Alt 1 (b)	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=\log_3\left(\frac{(x/9)^2}{\sqrt{x}}\right)$ or $2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\log_3 x-2\log_3 9-\log_3\sqrt{x}=\log_3\frac{x^2}{\sqrt{x}}+...$ Combines **two log terms in *x* correctly** to obtain a single log term		M1
	$\log_3\left(\frac{(x/9)^2}{\sqrt{x}}\right)=2$ or $\log_3\left(\frac{x^2}{\sqrt{x}}\right)=6$	Correct equation	A1
	$\left(\frac{(x/9)^2}{\sqrt{x}}\right)=3^2$ or $\left(\frac{x^2}{\sqrt{x}}\right)=3^6$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1

Alt 2 (b) Uses $x=3^y$	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\log_3\left(\frac{3^y}{9}\right)-\log_3 3^{\frac{y}{2}}=\log_3\left(\frac{3^{\frac{3y}{2}}}{81}\right)$ Combines logs correctly		M1
	$\log_3\left(\frac{3^{\frac{3y}{2}}}{81}\right)=2\Rightarrow y=4$	Correct value for y	A1
	$\log_3 x=4\Rightarrow x=3^4$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1

Leave blank

9. (i) Find the exact value of x for which

$$2\log_{10}(x-2)-\log_{10}(x+5)=0$$

(5)

(ii) Given

$$\log_p(4y+1)-\log_p(2y-2)=1 \qquad p>2,\ y>1$$

express y in terms of p.

(5)

Question Number	Scheme	Marks
9(i)	$2\log_{10}(x-2)-\log_{10}(x+5)=0 \Rightarrow \log_{10}(x-2)^2=\log_{10}(x+5)$	M1
	$\Rightarrow (x-2)^2=(x+5)$	M1
	$\Rightarrow x^2-5x-1=0$	A1
	$x=\dfrac{5\pm\sqrt{29}}{2} \Rightarrow x=\dfrac{5+\sqrt{29}}{2}$ only	M1,A1
		(5)
(ii)	$\log_p(4y+1)-\log_p(2y-2)=1 \Rightarrow \log_p\left(\dfrac{4y+1}{2y-2}\right)=\log_p p$	M1, M1
	$\Rightarrow \left(\dfrac{4y+1}{2y-2}\right)=p$	A1
	$\Rightarrow 4y+1=2py-2p \Rightarrow y=\dfrac{1+2p}{2p-4}$	M1A1
		(5) **(10 marks)**

(i)

M1 Use of the power law of logs

M1 For 'undoing' the logs by either setting $\log_{10}...=\log_{10}...$ or using the subtraction law and $0=\log_{10}1$

A1 A correct simplified quadratic $x^2-5x-1=0$

M1 A correct attempt to find a solution to a 3TQ of equivalent difficulty (ie no factors). Allow formula, completing the square and use of a calculator giving exact or decimal answers

A1 cso $\dfrac{5+\sqrt{29}}{2}$ or exact simplified equivalent without extra answers.

(ii)

M1 Use of subtraction (or addition) law of logs

M1 For using $1=\log_p p$ or equivalent in an attempt to get an equation not involving logs.

$\log_p(4y+1)-\log_p(2y-2)=1 \Rightarrow (4y+1)-(2y-2)=p$ implies this and scores M0 M1.

A1 A correct equation in p and y not involving logs. Accept $\left(\dfrac{4y+1}{2y-2}\right)=p^1$

M1 Score for an attempt to change the subject. This must include cross multiplication, collection of terms in y, followed by factorisation of the y term.

A1 cso $y=\dfrac{1+2p}{2p-4}$ or equivalent such as $y=\dfrac{-1-2p}{4-2p}$

Special cases in (i): Case 1 Allow the subtraction law either way around as the rhs of the equation will be 1

Case 2 $\log_{10}\dfrac{(x-2)^2}{(x+5)}=0 \Rightarrow \dfrac{(x-2)^2}{(x+5)}=0 \Rightarrow (x-2)^2=(x+5) \Rightarrow x^2-5x-1=0$

$\Rightarrow x=\dfrac{5+\sqrt{29}}{2}$ only will be awarded M1 M0 A1 M1 A0

Special cases in (ii): $\log_p(4y+1)-\log_p(2y-2)=1 \Rightarrow \dfrac{\log_p(4y+1)}{\log_p(2y-2)}=\log_p p \Rightarrow \left(\dfrac{4y+1}{2y-2}\right)=p^1$

$\Rightarrow 4y+1=2py-2p \Rightarrow y=\dfrac{1+2p}{2p-4}$ will be awarded M0 M1A1 M1 A0|
