MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Circular Measurements - C12 (Pearson Edexcel)

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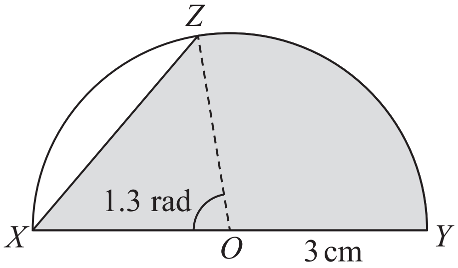

Diagram not drawn to scale

Figure 1

Figure 1 shows a semicircle with centre O and radius 3 cm. XY is the diameter of this semicircle. The point Z is on the circumference such that angle $XOZ = 1.3$ radians.
The shaded region enclosed by the chord XZ, the arc ZY and the diameter XY is a template for a badge.

Find, giving each answer to 3 significant figures,

(a) the length of the chord XZ,

(2)

(b) the perimeter of the template $XZYX$,

(4)

(c) the area of the template.

(4)

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Question Number	Scheme	Marks
10.(a)	$XZ^2 = 3^2 + 3^2 - 2\times3\times3\cos1.3$, or $\sin 0.65 = \frac{x}{3}$ *so* $XZ = 2\times x$	M1
	$XZ = 3.63$	A1 **[2]**
(b)	Arc length $ZY = 3\times\theta$,= 3 × (π – 1.3) (= 5.52 / 5.53)	M1, A1
	Perimeter = 3 + 3 + arc ZY + chord XZ = 15.2 (cm)	dM1 A1 **[4]**
(c)	Area of triangle $OXZ = \frac{1}{2}\times3\times3\times\sin1.3$ (=4.34)	M1
	Area of sector is $\frac{1}{2}r^2\theta = \frac{1}{2}\times3^2\times(\pi-1.3)$ (= 8.28 / 8.29)	M1
	Total area is $\frac{1}{2}\times3^2\times(\pi-1.3)+\frac{1}{2}\times3\times3\times\sin1.3$	dM1
	= 12.6 (cm^2)	A1 **[4]**
		10 marks

Notes

(a)

M1: Uses cosine rule – must be correct. Allow $XZ^2 = 3^2 + 3^2 - 2\times3\times3\cos1.3$, for the M1

Or splits into right angled triangles correctly, uses sin 0.65 and then doubles the result

Uses angles in a triangle rule with the sine rule to find the required side. Eg $\frac{x}{\sin1.3} = \frac{3}{\sin0.92}$

A1: awrt 3.63

(b)

M1: Arc length formula *r θ* with r = 3 and $\theta = 1.3, (\pi - 1.3)$ or $(2\pi - 1.3)$ If decimals are seen accept 1.8 or 5.0

If the degree formula is being used look for $\frac{\theta}{360}\times2\pi r$ with $\theta = 74° - 75°$ or $\theta = 105° - 106°$

A1: Uses arc length formula with a correct angle. It does not need to be processed

Allow $3(\pi - 1.3), 3\times1.84$, awrt 5.52 / 5.53 In degrees look for the minimum accuracy of $\frac{105.5}{360}\times2\pi\times3$

dM1: Complete method for perimeter. It is dependent upon the previous M. Look for $6+(a)+\text{arc length}$

A1: awrt15.2 (cm) – you do not need to see units

(c)

M1: Uses area formula for triangle correctly. If $\frac{1}{2}bh$ is used it must be the correct combinations found using a correct method.

M1: Uses the formula $\frac{1}{2}r^2\theta$ to find the area of the correct sector. There must be some valid attempt to use the correct angle. Allow as a minimum awrt 1.8 radians $(3.1-1.3)$

dM1: Adds two correct area formulae together. Both M's must have been awarded

A1: Accept awrt 12.6 (do not need units)

Alt (c)

M1: Attempts to find the area of the segment $\frac{1}{2}\times3^2(1.3-\sin1.3)$

M1: Attempts area of semi circle **along with** the area of segment

dM1: Finds area of the semi circle - segment $\frac{\pi\times3^2}{2}-\frac{1}{2}\times3^2(1.3-\sin1.3)$

A1: awrt 12.6

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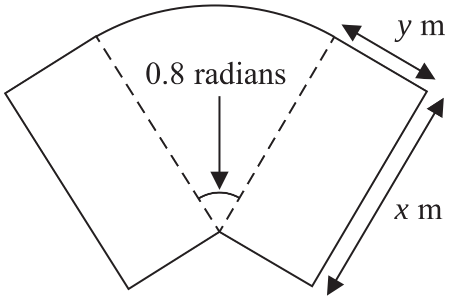

Figure 2

Figure 2 shows a plan for a garden.
The garden consists of two identical rectangles of width y m and length x m, joined to a sector of a circle with radius x m and angle 0.8 radians, as shown in Figure 2.

The area of the garden is 60 m^2.

(a) Show that the perimeter, P m, of the garden is given by

$$P = 2x + \frac{120}{x}$$

(5)

(b) Use calculus to find the exact minimum value for P, giving your answer in the form $a\sqrt{b}$, where a and b are integers.

(4)

(c) Justify that the value of P found in part (b) is the minimum.

(2)

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Question Number	Scheme	Notes	Marks
15(a)	$(\text{Arc length} =)\,0.8x$	Correct expression	B1
	$P = 2x + 4y + 0.8x$	$P = \alpha x + \beta y + "0.8x", \quad \alpha, \beta \neq 0$	M1
	This may be implied by e.g. $P = 2x + 4(\text{their } y) + 0.8x$		
	$2xy + \frac{1}{2}(0.8)x^2 = 60$	Correct equation for the area	B1
	$y = \frac{60 - 0.4x^2}{2x} \Rightarrow P = 4\left(\frac{60 - 0.4x^2}{2x}\right) + 2.8x$	Makes y the subject and substitutes	M1
	$P = \frac{120}{x} + 2x$ *	Obtains printed answer with no errors with $P = \ldots$ or Perimeter $= \ldots$ appearing at some point.	A1*
	Note that it is sufficient to go from $P = 4\left(\frac{60 - 0.4x^2}{2x}\right) + 2.8x$ to $P = \frac{120}{x} + 2x$ *		
			(5)

15(b)	**Mark (b) and (c) together** **Allow e.g.** $\frac{dy}{dx}$ **for** $\frac{dP}{dx}$ **and/or** $\frac{d^2y}{dx^2}$ **for** $\frac{d^2P}{dx^2}$		
	$\frac{dP}{dx}=2-\frac{120}{x^2}$	Correct derivative	B1
	$2-\frac{120}{x^2}=0\Rightarrow x=\sqrt{60}$	$\frac{dP}{dx}=0$ and solves for x. Must be fully correct algebra for their $\frac{dP}{dx}=0$ which is solvable.	M1
	$P=\frac{120}{\sqrt{60}}+2\sqrt{60}$	Substitutes into P, a **positive** x which has come from an attempt to solve their $\frac{dP}{dx}=0$	M1
	$P=4\sqrt{60}$ or $8\sqrt{15}$ or $\sqrt{960}$	Correct exact answer. Cso.	A1
	Note that if $\frac{dP}{dx}=2+\frac{120}{x^2}$ **is obtained, this could score a maximum of B0M0M1A0 if a positive value of** x **is substituted into** P**.**		
			(4)
(c)			
	$\left(\frac{d^2P}{dx^2}=\right)\frac{240}{x^3}=\frac{240}{\left(\sqrt{60}\right)^3}$	Attempts the second derivative $x^n\to x^{n-1}$ seen at least once (allow k → 0 as evidence) and then substitutes at least one **positive** value of x from their $\frac{dP}{dx}=0$ **or** makes reference to the sign of the second derivative provided they have a positive x.	M1
	$\left(\frac{d^2P}{dx^2}=\right)\frac{240}{\left(\sqrt{60}\right)^3}\Rightarrow\frac{d^2P}{dx^2}>0\therefore$ minimum Requires a **correct second derivative** and the **correct value of** x. There must be a reference to the sign of the second derivative. If x is substituted and then $\frac{d^2P}{dx^2}$ is evaluated incorrectly allow this mark if the other conditions are met. If x is not substituted then the reference to $\frac{d^2P}{dx^2}$ being positive must also include a reference to the fact that x is positive.		A1
	Allow alternatives e.g. considers **values** of P either side of $\sqrt{60}$ or **values** of $\frac{dP}{dx}$ either side of $\sqrt{60}$ can score M1 and then A1 if a full **reason and conclusion** is given.		
			(2)
			Total 11

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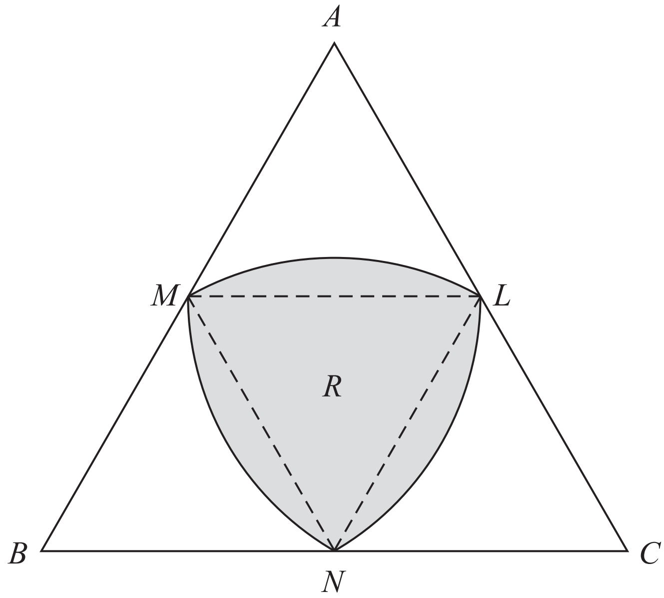

Figure 5

Figure 5 shows the design for a logo.

The logo is in the shape of an equilateral triangle ABC of side length $2r$ cm, where r is a constant.

The points L, M and N are the midpoints of sides AC, AB and BC respectively.

The shaded section R, of the logo, is bounded by three curves MN, NL and LM.

The curve MN is the arc of a circle centre L, radius r cm.

The curve NL is the arc of a circle centre M, radius r cm.

The curve LM is the arc of a circle centre N, radius r cm.

Find, in cm^2, the area of R. Give your answer in the form kr^2, where k is an exact constant to be determined.

(5)

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<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>15</td><td colspan="2">Area of triangle = $\frac{1}{2}\times(2r)^2\sin\left(\frac{\pi}{3}\text{ or }60\right)$ or $\frac{1}{2}\times(r)^2\sin\left(\frac{\pi}{3}\text{ or }60\right)$
Correct method for the area of either triangle. Ignore any reference to which triangle they are finding the area of.</td><td>M1</td></tr>
<tr><td></td><td>Area of sector = $\frac{1}{2}\times r^2\times\frac{\pi}{3}$</td><td>Use of the sector formula $\frac{1}{2}r^2\theta$ with $\theta=\frac{\pi}{3}$ which may be embedded within a segment</td><td>M1</td></tr>
<tr><td></td><td colspan="2">Area R = Sector + 2 Segments $=\frac{1}{2}r^2\times\frac{\pi}{3}+2\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)$
Area R = Triangle + 3 Segments $=\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}+3\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)$
Area R = 3 Sectors − 2 Triangles $=3\times\frac{1}{2}r^2\times\frac{\pi}{3}-2\times\left(\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)$
Area R = Big triangle − 3 White bits
$=\frac{1}{2}\times(2r)^2\frac{\sqrt{3}}{2}-3\times\left(\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}-\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)\right)$
M1: A fully correct method (may be implied by a final answer of awrt $0.705r^2$)
A1: Correct exact expression - for this to be scored $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ must be seen</td><td>M1A1</td></tr>
<tr><td></td><td>$=\frac{1}{2}\pi r^2-\frac{\sqrt{3}}{2}r^2=r^2\left(\frac{1}{2}\pi-\frac{\sqrt{3}}{2}\right)$</td><td>Cso (Allow $\frac{r^2}{2}\left(\pi-\sqrt{3}\right)$ or any exact equivalent with r^2 taken out as a common factor)</td><td>A1</td></tr>
<tr><td></td><td colspan="2"></td><td>(5 marks)</td></tr>
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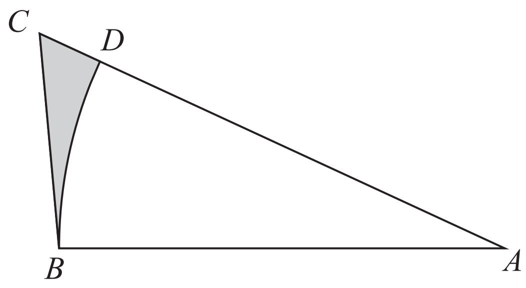

Figure 2

Figure 2 shows a sketch of a design for a triangular garden *ABC*.

The garden has sides *BA* with length 10 m, *BC* with length 6 m and *CA* with length 12 m.

The point *D* lies on *AC* such that *BD* is an arc of the circle centre *A*, radius 10 m.

A flowerbed *BCD* is shown shaded in Figure 2.

(a) Find the size of angle *BAC*, in radians, to 4 decimal places.
(2)

(b) Find the perimeter of the flowerbed *BCD*, in m, to 2 decimal places.
(3)

(c) Find the area of the flowerbed *BCD*, in m^2, to 2 decimal places.
(4)

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Question Number	Scheme	Marks
6(a)	$\cos\angle BAC = \frac{12^2+10^2-6^2}{2\times12\times10} \Rightarrow \angle BAC = 0.5223$	M1A1
		(2)
(b)	Arc $BD = r\theta = 10\times0.5223$	M1
	Perimeter = 6+2+ $10\times0.5223=13.22$ (m)	dM1,A1
		(3)
(c)	Area of sector $BAD = \frac{1}{2}r^2\theta = \frac{1}{2}\times10^2\times0.5223 \quad (=26.116)$	M1
	Area of triangle $ABC\,\frac{1}{2}ab\sin C = \frac{1}{2}\times12\times10\times\sin0.5223 \quad (=29.932)$	M1
	Area of flowerbed BCD = $\frac{1}{2}\times12\times10\times\sin0.5223$ - $\frac{1}{2}\times10^2\times0.5223$	dM1
	= 3.81 / 3.82 (m^2)	A1
		(4)
		(9 marks)

(a)

M1 Attempts use of the formula $6^2=10^2+12^2-2\times10\times12\cos A$ or $\cos\angle BAC = \frac{12^2+10^2-6^2}{2\times12\times10}$

The sides must be in the correct "position" within the formula. Condone different notation Eg. θ

A1 $\angle BAC =$ awrt 0.5223 The angle in degrees (awrt $29.9°$) is A0

(b)

M1 Attempts arc formula: In radians uses Arc $BD = r\theta = 10\times\text{"}0.5223\text{"}$

In degrees uses Arc $BD = \frac{\theta}{360}\times2\pi r = \frac{\text{"}29.9\text{"}}{360}\times2\pi\times10$

dM1 Dependent upon the arc formula having been used. It is for calculating the perimeter as 8 + arc length.

A1 Perimeter = awrt 13.22 (m)

(c)

M1 Attempts area of sector formula: Area of sector $BAD = \frac{1}{2}r^2\theta = \frac{1}{2}\times10^2\times\text{"}0.5223\text{"}$

In degrees uses Area of sector $BAD = \frac{\theta}{360}\times\pi r^2 = \frac{\text{"}29.9\text{"}}{360}\times\pi\times10^2$

M1 Attempts area of triangle formula: Area of triangle $ABC = \frac{1}{2}ab\sin C = \frac{1}{2}\times12\times10\times\sin\text{"}0.5223\text{"}$

You may see Herons formula used with $S = \frac{10+6+12}{2} = (14)$ and $A = \sqrt{S(S-10)(S-6)(S-12)}$

Watch for other methods including the calculation of a perpendicular.

dM1 Dependent upon both correct formulae. It is scored for finding area of triangle - area of sector

A1 Allow awrt 3.81 or 3.82 (m^2)

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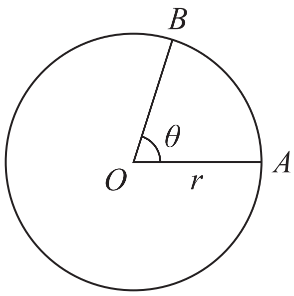

Figure 3

Figure 3 shows a circle with centre O and radius r cm.

The points A and B lie on the circumference of this circle.

The minor arc AB subtends an angle θ radians at O, as shown in Figure 3.

Given the length of minor arc AB is 6 cm and the area of minor sector OAB is 20 cm^2,

(a) write down two different equations in r and θ.

(2)

(b) Hence find the value of r and the value of θ.

(4)

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Question Number	Scheme	Marks
8.(a)	$r\theta = 6$ and $\frac{1}{2}r^2\theta = 20$	B1 B1
		[2]
(b)	Substitute $r\theta = 6$ into $\frac{1}{2}r^2\theta = 20 \Rightarrow \frac{1}{2}\times 6r = 20$	M1
	$\Rightarrow r = \frac{20}{3}$	A1
	Substitutes $r = \frac{20}{3}$ in $r\theta = 6 \Rightarrow \theta = \frac{9}{10}$	dM1A1
		[4]
		(6 marks)

This may be marked as one complete question. Eg they may just give the equations $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in (a)
Don't penalise this sort of error.

(a)

B1 Either $r\theta = 6$ or $\frac{1}{2}r^2\theta = 20$ (or exact equivalents)

Allow $\frac{\theta}{2\pi}\times 2\pi r = 6$ or $\frac{\theta}{2\pi}\times \pi r^2 = 20$ but not $\frac{\theta}{360}\times 2\pi r = 6$ or $\frac{\theta}{360}\times \pi r^2 = 20$

B1 Both $r\theta = 6$ and $\frac{1}{2}r^2\theta = 20$ (or exact equivalents)

Allow $\frac{\theta}{2\pi}\times 2\pi r = 6$ and $\frac{\theta}{2\pi}\times \pi r^2 = 20$ but not $\frac{\theta}{360}\times 2\pi r = 6$ and $\frac{\theta}{360}\times \pi r^2 = 20$

(b)

M1 Combines two equations in r and θ producing an equation in one unknown.

A1 $r = \frac{20}{3}$ or $\theta = \frac{9}{10}$ or exact equivalents.

You may just see answers following correct equations. This is fine for all the marks

dM1 This is dependent upon having started with two equations with correct expressions in r and θ
Look for $..r\theta = ...$ and $..r^2\theta = ...$.
It is awarded for correctly substituting their value of r or θ into one of the equations to find the second unknown.

A1 $r = \frac{20}{3}$ and $\theta = \frac{9}{10}$ or exact equivalents. Condone $6.\dot{6}$ for $\frac{20}{3}$ Do not allow 6.67