# MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

## Chapters:

### Differentiation - C3 (Pearson Edexcel)

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**1.** Given $y = 2x(3x - 1)^5$,

(a) find $\frac{dy}{dx}$, giving your answer as a single fully factorised expression.

**(4)**

(b) Hence find the set of values of $x$ for which $\frac{dy}{dx} \leqslant 0$

**(2)**

| Question Number | Scheme | Marks |
|---|---|---|
| **1.(a)** | $y = 2x(3x-1)^5 \Rightarrow \dfrac{dy}{dx} = 2(3x-1)^5 + 30x(3x-1)^4$ | M1A1 |
| | $\Rightarrow \left(\dfrac{dy}{dx}\right) = 2(3x-1)^4\{(3x-1)+15x\} = 2(3x-1)^4(18x-1)$ | M1A1 |
| | | **(4)** |
| **(b)** | $\dfrac{dy}{dx} \leqslant 0 \Rightarrow 2(3x-1)^4(18x-1) \leqslant 0 \Rightarrow x \leqslant \dfrac{1}{18} \quad x = \dfrac{1}{3}$ | B1ft, B1 |
| | | **(2)** |
| | | **(6 marks)** |

**This may be marked as one complete question**

(a)

**M1**: Uses the product rule $vu'+uv'$ with $u = 2x$ and $v = (3x-1)^5$ or vice versa to achieve an expression of the form $A(3x-1)^5 + Bx(3x-1)^4, \quad A, B > 0$

Condone slips on the $(3x-1)$ and $2x$ terms but misreads on the question must be of equivalent difficulty. If in doubt use review.

Eg: $y = 2x(3x+1)^5 \Rightarrow \dfrac{dy}{dx} = 2(3x+1)^5 + 30x(3x+1)^4$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y = 2x(3x+1)^{15} \Rightarrow \dfrac{dy}{dx} = 2(3x+1)^{15} + 90x(3x+1)^{14}$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y = 2(3x+1)^5 \Rightarrow \dfrac{dy}{dx} = 30(3x+1)^4$ is 0000 even if attempted using the product rule (as it is easier)

**A1**: A correct un-simplified expression. You may never see the lhs which is fine for all marks.

**M1:** Scored for taking a common factor of $(3x-1)^4$ out of $A(3x-1)^5 \pm Bx^n(3x-1)^4$ where $n=1$ or 2,to reach a form $(3x-1)^4\{........\}$ You may condone one slip in the $\{........\}$

Alternatively they take out a common factor of $2(3x-1)^4$ which can be scored in the same way

Example of one slip $2(3x-1)^5 + 30x(3x-1)^4 = (3x-1)^4\{(3x-1)+30x\}$

If a different form is reached, see examples above, it is for equivalent work.

**A1:** Achieves a fully factorised simplified form $2(3x-1)^4(18x-1)$ which may be awarded in (b)

(b)

**B1ft: For a final answer** of either $x \leqslant \dfrac{1}{18}$ or $x = \dfrac{1}{3}$ Condone $x \leqslant \dfrac{2}{36}$ $x \leq 0.0\dot{5}$ $x = 0.\dot{3}$

Do not allow $x = \dfrac{1}{3}$ if followed by $x \leqslant \dfrac{1}{3}$ Follow through on a linear factor of $(Ax+B) \leqslant 0 \Rightarrow x...$ where $A, B \neq 0$ . Watch for negative $A$'s where the inequality would reverse.

It may be awarded within an equality such as $\dfrac{1}{3} \leqslant x \leqslant \dfrac{1}{18}$

**B1:** **For a final answer** of $x \leqslant \dfrac{1}{18}$ oe (and) $x = \dfrac{1}{3}$ oe with no other solutions. Ignore any references to and/or here. Misreads can score these marks

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**4.**

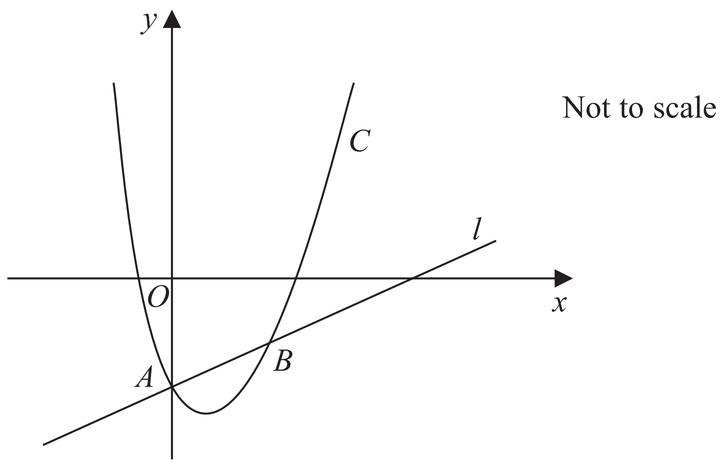


**Figure 1**

Figure 1 shows a sketch of part of the curve $C$ with equation

$$y = e^{-2x} + x^2 - 3$$

The curve $C$ crosses the $y$-axis at the point $A$.

The line $l$ is the normal to $C$ at the point $A$.

(a) Find the equation of $l$, writing your answer in the form $y = mx + c$, where $m$ and $c$ are constants.
**(5)**

The line $l$ meets $C$ again at the point $B$, as shown in Figure 1.

(b) Show that the $x$ coordinate of $B$ is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

**(2)**

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

(c) find $x_2$ and $x_3$ to 3 decimal places.
**(2)**

| Question Number | Scheme | Marks |
|---|---|---|
| **4.(a)** | $\frac{dy}{dx} = -2e^{-2x} + 2x$ | M1A1 |
| | At $x = 0$ $\frac{dy}{dx} = -2 \Rightarrow \frac{dx}{dy} = \frac{1}{2}$ | M1 |
| | Equation of normal is $y - (-2) = \frac{1}{2}(x - 0) \Rightarrow y = \frac{1}{2}x - 2$ | M1 A1 |
| | | **(5)** |
| **(b)** | $y = e^{-2x} + x^2 - 3$ meets $y = \frac{1}{2}x - 2$ when $e^{-2x} + x^2 - 3 = "\frac{1}{2}x - 2"$ | |
| | $x^2 = 1 + \frac{1}{2}x - e^{-2x}$ | M1 |
| | $x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$ * | A1* |
| | | **(2)** |
| **(c)** | $x_2 = \sqrt{1 + 0.5 - e^{-2}}$ | M1 |
| | $x_2 = 1.168,\ x_3 = 1.220$ | A1 |
| | | **(2)** |
| | | **(9 marks)** |

(a)

**M1:** Attempts to differentiate with $e^{-2x} \to Ae^{-2x}$ with any non -zero $A$, even 1.

Watch for $e^{-2x} \to Ae^{2x}$ which is M0 A0

**A1:** $\frac{dy}{dx} = -2e^{-2x} + 2x$

**M1:** A correct method of finding the gradient of the normal at $x = 0$

To score this the candidate must find the negative reciprocal of $\left.\frac{dy}{dx}\right|_{x=0}$

So for example candidates who find $\frac{dy}{dx} = e^{-2x} + 2x$ should be using a gradient of $-1$

Candidates who write down $\frac{dy}{dx} = -2$ (from their calculators?) have an opportunity to score this mark and the next.

**M1:** An attempt at the equation of the normal at $(0,-2)$

To score this mark the candidate must be using the point $(0,-2)$ and a gradient that has been **changed from** $\left.\frac{dy}{dx}\right|_{x=0}$

Look for $y - (-2) = \textit{changed}\left.\frac{dy}{dx}\right|_{x=0}(x - 0)$ or $y = mx - 2$ where $m = \textit{changed}\left.\frac{dy}{dx}\right|_{x=0}$

If there is an attempt using $y = mx + c$ then it must proceed using $(0,-2)$ with $m = \textit{changed}\left.\frac{dy}{dx}\right|_{x=0}$

**A1:** $y=\frac{1}{2}x-2$ cso with as well as showing the correct differentiation.

So reaching $y=\frac{1}{2}x-2$ from $\frac{dy}{dx}=-2e^{2x}+2x$ is A0

If it is not simplified (or written in the required form) you may award this if $y=\frac{1}{2}x-2$ is seen in part (b)

(b)

**M1:** Equates $y=e^{-2x}+x^2-3$ and their $y=mx+c, m\neq 0$ and proceeds to $x^2=...$
Condone an attempt for this M mark where the candidate uses an adapted $y=mx+c$ in an attempt to get the printed answer.

**A1*:** Proceeds to $x=\sqrt{1+\frac{1}{2}x-e^{-2x}}$. It is a printed answer but you may accept a different order $x=\sqrt{1-e^{-2x}+\frac{1}{2}x}$

For this mark, the candidate must start with a normal equation of $y=\frac{1}{2}x-2$ oe found in (a). It can be awarded when the candidate finds the equation incorrectly, for example from $\frac{dy}{dx}=-2e^{2x}+2x$

(c)

**M1:** Sub $x_1=1$ in $x=\sqrt{1+\frac{1}{2}x-e^{-2x}}$ to find $x_2$. May be implied by $\sqrt{1+0.5-e^{-2}}$ oe or awrt 1.17

**A1:** $x_2=\text{awrt } 1.168, x_3=\text{awrt } 1.220$ 3dp. Condone 1.22 for $x_3$

Mark these in the order given, the subscripts are not required and incorrect ones may be ignored.

Leave blank

**7.** The curve $C$ has equation $y = \dfrac{\ln(x^2 + 1)}{x^2 + 1}, \quad x \in \mathbb{R}$

(a) Find $\dfrac{dy}{dx}$ as a single fraction, simplifying your answer.
**(3)**

(b) Hence find the exact coordinates of the stationary points of $C$.
**(6)**

| Question Number | Scheme | Marks |
|---|---|---|
| **7.(a)** | Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{\ln(x^2+1)}{x^2+1}$ with $u=\ln(x^2+1)$ and $v=x^2+1$ | |
| | $\frac{dy}{dx}=\frac{(x^2+1)\times\frac{2x}{x^2+1}-2x\ln(x^2+1)}{(x^2+1)^2}$ | M1 A1 |
| | $\frac{dy}{dx}=\frac{2x-2x\ln(x^2+1)}{(x^2+1)^2}$ | A1 |
| | | **(3)** |
| **(b)** | Sets $2x-2x\ln(x^2+1)=0$ | M1 |
| | $2x(1-\ln(x^2+1))=0\Rightarrow x=\pm\sqrt{e-1},$ | M1,A1 |
| | Sub $x=\pm\sqrt{e-1}, 0$ into $f(x)=\frac{\ln(x^2+1)}{x^2+1}$ | dM1 |
| | Stationary points $\left(\sqrt{e-1},\frac{1}{e}\right),\left(-\sqrt{e-1},\frac{1}{e}\right),\underline{\underline{(0,0)}}$ | A1 $\underline{\underline{\text{B1}}}$ |
| | | **(6)** |
| | | **(9 marks)** |

(a)

**M1:** Attempts the quotient or product rule to achieve an expression in the correct form

Using the quotient rule achieves an expression of the form $\frac{dy}{dx}=\frac{(x^2+1)\times\frac{\dots}{x^2+1}-2x\ln(x^2+1)}{(x^2+1)^2}$

or the form $\frac{dy}{dx}=\frac{\dots-2x\ln(x^2+1)}{(x^2+1)^2}$ where $\dots=A$ or $Ax$

or using the product rule achieves and an expression $\frac{dy}{dx}=(x^2+1)^{-1}\times\frac{\dots}{x^2+1}-2x(x^2+1)^{-2}\ln(x^2+1)$

You may condone the omission of brackets .........especially on the denominator

**A1:** A correct un-simplified expression for $\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{(x^2+1)\times\frac{2x}{x^2+1}-2x\ln(x^2+1)}{(x^2+1)^2} \text{ or } \frac{dy}{dx}=(x^2+1)^{-1}\times\frac{2x}{x^2+1}-2x(x^2+1)^{-2}\ln(x^2+1)$$

**A1:** $\frac{dy}{dx}=\frac{2x-2x\ln(x^2+1)}{(x^2+1)^2}$ or exact simplified equivalent such as $\frac{dy}{dx}=\frac{2x(1-\ln(x^2+1))}{(x^2+1)^2}$ .

Condone $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-\ln\left(x^2+1\right)2x}{\left(x^2+1\right)^2}$ which may be a little ambiguous. The lhs $\frac{\mathrm{d}y}{\mathrm{d}x}=$ does not need to be seen. You may assume from the demand in the question that is what they are finding.
ISW can be applied here.

(b)

**M1:** Sets the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$, which must contain at least two terms, equal to 0

**M1:** For solving an equation of the form $\ln(x^2+1)=k, \quad k>0$ to get at least one non- zero value of $x$. Accept decimal answers. $x = awrt \pm 1.31$ The equation must be legitimately obtained from a numerator = 0

**A1:** Both $x=\pm\sqrt{\mathrm{e}-1}$ scored from ± a correct numerator Condone $x=\pm\sqrt{\mathrm{e}^1-1}$

**dM1**: Substitutes any of their non zero solutions to $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into $\mathrm{f}(x)=\frac{\ln\left(x^2+1\right)}{x^2+1}$ to find at least one '$y$' value. It is dependent upon both previous M's

**A1:** Both $\left(\sqrt{\mathrm{e}-1},\frac{1}{\mathrm{e}}\right),\left(-\sqrt{\mathrm{e}-1},\frac{1}{\mathrm{e}}\right)$ oe or the equivalent with $x=..., y=...$ ln e must be simplified

Condone $\left(\sqrt{\mathrm{e}^1-1},\frac{1}{\mathrm{e}^1}\right),\left(-\sqrt{\mathrm{e}^1-1},\frac{1}{\mathrm{e}^1}\right)$ but the $y$ coordinates must be simplified as shown.

Condone $\left(\pm\sqrt{\mathrm{e}-1},\frac{1}{\mathrm{e}}\right)$ Withhold this mark if there are extra solutions to these apart from (0,0)

It can only be awarded from ± a correct numerator

**B1:** $(0,0)$ or the equivalent $x=0, y=0$

Notes:

(1) A candidate can "recover" and score all marks in (b) when they have an incorrect denominator in part (a) or a numerator the wrong way around in (a)

(2) A candidate who differentiates $\ln\left(x^2+1\right)\to\frac{1}{x^2+1}$ will probably only score (a) 100 (b) 100000

(3) A candidate who has $\frac{vu'+uv'}{v^2}$ cannot score anything more than (a) 000 (b)100001 as they would have $k<0$

(4) A candidate who attempts the product rule to get $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2+1\right)^{-1}\times\frac{1}{x^2+1}-\left(x^2+1\right)^{-2}\ln\left(x^2+1\right)=\frac{1-\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$

can score (a) 000 (b) 110100 even though they may obtain the correct non zero coordinates.

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**8.** (a) By writing $\sec\theta = \dfrac{1}{\cos\theta}$, show that $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\sec\theta) = \sec\theta\tan\theta$

**(2)**

(b) Given that

$$x = \mathrm{e}^{\sec y} \qquad x > \mathrm{e}, \quad 0 < y < \frac{\pi}{2}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x\sqrt{\mathrm{g}(x)}}, \qquad x > \mathrm{e}$$

where $\mathrm{g}(x)$ is a function of $\ln x$.

**(5)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

| Question Number | Scheme | Marks |
|---|---|---|
| **8(a)** | $\frac{d}{d\theta}(\sec\theta) = \frac{d}{d\theta}(\cos\theta)^{-1} = -1\times(\cos\theta)^{-2}\times -\sin\theta$ | M1 |
| | $= \frac{1}{\cos\theta}\times\frac{\sin\theta}{\cos\theta}$ | |
| | $= \sec\theta\tan\theta$ | A1* |
| | | **(2)** |
| **(b)** | $x = e^{\sec y} \Rightarrow \frac{dx}{dy} = e^{\sec y}\times\sec y\tan y$ oe | M1A1 |
| | $\Rightarrow \frac{dy}{dx} = \frac{1}{e^{\sec y}\times\sec y\tan y}$ | M1 |
| | Uses $1+\tan^2 y = \sec^2 y$ with $\sec y = \ln x \Rightarrow \tan y = \sqrt{(\ln x)^2 - 1}$ | M1 |
| | $\Rightarrow \frac{dy}{dx} = \frac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}} = \frac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe | A1 |
| | | **(5)** |
| | | **(7 marks)** |
| **Alt (b)** | $\ln x = \sec y \Rightarrow \frac{1}{x}\frac{dx}{dy} = \sec y\tan y$ | **M1A1** |
| | $\Rightarrow \frac{dy}{dx} = \frac{1}{x\times\sec y\tan y}$ | **M1** |
| | Uses $1+\tan^2 y = \sec^2 y$ and $\sec y = \ln x \Rightarrow \tan y = \sqrt{(\ln x)^2-1}$ | **M1** |
| | $\Rightarrow \frac{dy}{dx} = \frac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}} = \frac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe | **A1** |
| | | **(5)** |

(a)

**M1:** Uses the chain rule to get $\pm 1\times(\cos\theta)^{-2}\times\sin\theta$

Alternatively uses the quotient rule to get $\dfrac{\cos\theta\times 0\pm 1\times\sin\theta}{\cos^2\theta}$ condoning the denominator as $\cos\theta^2$

When applying the quotient rule it is very difficult to see if the correct rule has been used. So only withhold this mark if an incorrect rule is quoted.

**A1*:** Completes proof with no errors (see below *) and shows line $\dfrac{1}{\cos\theta}\times\dfrac{\sin\theta}{\cos\theta}$, $\dfrac{\tan\theta}{\cos\theta}$ or $\dfrac{\sin\theta}{\cos\theta\times\cos\theta}$ before the given answer. The notation should be correct so do not allow if they start $y=\sec\theta\Rightarrow\dfrac{dy}{dx}=\sec\theta\tan\theta$

* You do not need to see $\dfrac{d}{d\theta}(\sec\theta)=...$ or $\dfrac{dy}{d\theta}$ anywhere in the solution

(b)

M1 Differentiates to get the rhs as $e^{\sec y}\times...$

A1 Completely correct differential inc the lhs $\dfrac{dx}{dy}=e^{\sec y}\times\sec y\tan y$

M1 Inverts **their** $\dfrac{dx}{dy}$ to get $\dfrac{dy}{dx}$.

The variable used **must be** consistent. Eg $\dfrac{dx}{dy}=e^{\sec y}\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{e^{\sec x}}$ is M0

M1 For attempting to use $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x$

(You may condone $\ln x^2\to 2\ln x$ for the method mark)

It may be implied by $\tan y=\sqrt{\pm(\ln x)^2\pm 1}$ They must have a term in $\tan y$ to score this.

A valid alternative would be attempting to use $1+\cot^2 y=\mathrm{cosec}^2 y$ with $\mathrm{cosec}\, y=\dfrac{1}{\sqrt{1-\frac{1}{\ln^2 x}}}$ oe

A1 $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ or exact equivalents such as $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{\ln^4 x-\ln^2 x}}$

Do not isw here. Withhold this mark if candidate then writes down $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{4(\ln x)-2(\ln x)}}$

Also watch for candidates who write $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{\ln x^4-\ln x^2}}$ which is incorrect (without the brackets)

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**8.**

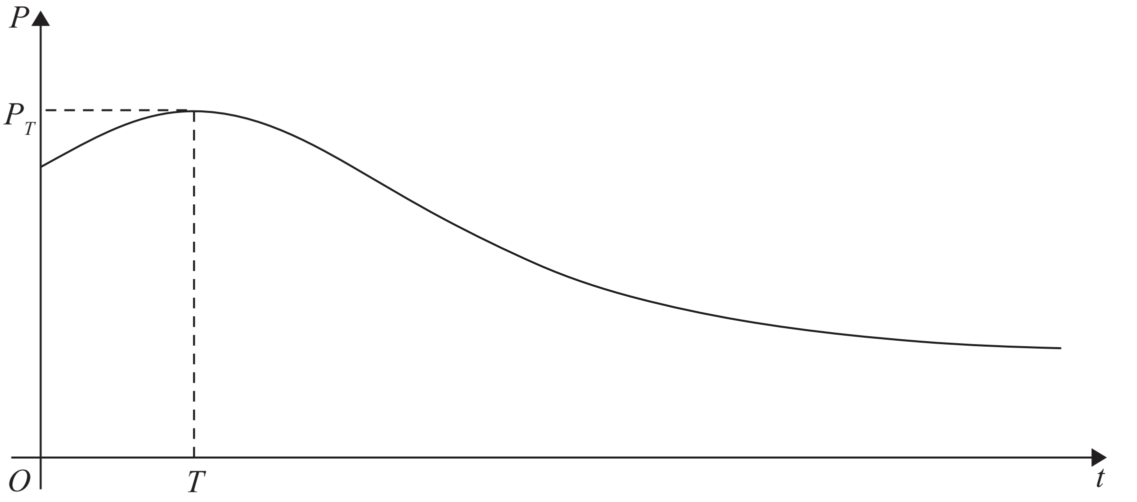


**Figure 3**

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1 + 3e^{-0.9t}} + 40, \qquad t \in \mathbb{R}, t \geqslant 0$$

where $P$ is the number of rabbits, $t$ years after they were introduced onto the island.

A sketch of the graph of $P$ against $t$ is shown in Figure 3.

(a) Calculate the number of rabbits that were introduced onto the island.

**(1)**

(b) Find $\frac{dP}{dt}$

**(3)**

The number of rabbits initially increases, reaching a maximum value $P_T$ when $t = T$

(c) Using your answer from part (b), calculate

(i) the value of $T$ to 2 decimal places,

(ii) the value of $P_T$ to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(4)**

For $t > T$, the number of rabbits decreases, as shown in Figure 3, but never falls below $k$, where $k$ is a positive constant.

(d) Use the model to state the maximum value of $k$.

**(1)**

| Question Number | Scheme | Marks |
|---|---|---|
| **8 (a)** | $P_0 = \frac{100}{1+3} + 40 = 65$ | B1 |
| | | (1) |
| | $\frac{d}{dt}e^{kt} = Ce^{kt}$ | M1 |
| **(b)** | $\frac{dP}{dt} = \frac{\left(1+3e^{-0.9t}\right)\times -10e^{-0.1t} - 100e^{-0.1t} \times -2.7e^{-0.9t}}{\left(1+3e^{-0.9t}\right)^2}$ | M1 A1 |
| | | (3) |
| **(c)(i)** | At maximum $-10e^{-0.1t} - 30e^{-0.1t} \times e^{-0.9t} + 270e^{-0.1t} \times e^{-0.9t} = 0$ | |
| | $e^{-0.1t}\left(-10 + 240e^{-0.9t}\right) = 0$ | |
| | $e^{-0.9t} = \frac{10}{240}$ oe $e^{0.9t} = 24$ | M1 |
| | $-0.9t = \ln\left(\frac{1}{24}\right) \Rightarrow t = \frac{10}{9}\ln(24) = 3.53$ | M1, A1 |
| **(c) (ii)** | Sub $t = 3.53 \Rightarrow P_T = 102$ | A1 |
| | | (4) |
| **(d)** | 40 | B1 |
| | | (1) |
| | | **9 marks** |

(a)

B1 $\left(P_0 =\right) 65$

(b)

M1 For sight of $\frac{d}{dt}e^{kt} = Ce^{kt}$ (Allow $C$ =1)This may be within an incorrect product or quotient rule

M1 Scored for a full application of the quotient rule. If the formula is quoted it should be correct.

The denominator should be present even when the correct formula has been quoted.

In cases where a formula has not been quoted it is very difficult to judge that a correct formula has been used (due to the signs between the terms). So…………….

if the formula has not been quoted look for the **order** of the terms $\frac{(1+3e^{-0.9t})\times pe^{-0.1t} - qe^{-0.1t}\times e^{-0.9t}}{(1+3e^{-0.9t})^2}$

$$\frac{(1+3e^{-0.9t})\times pe^{-0.1t} + qe^{-0.1t}\times e^{-0.9t}}{(1+3e^{-0.9t})^2}$$

For the product rule. Look for $ae^{-0.1t}\left(1+3e^{-0.9t}\right)^{-1} \pm be^{-0.1t}e^{-0.9t}\left(1+3e^{-0.9t}\right)^{-2}$ either way around

Penalise if an incorrect formula is quoted . Condone missing brackets in both cases.

A1 A correct **unsimplified** answer.

Eg using quotient rule $\left(\frac{dP}{dt}\right) = \frac{-10e^{-0.1t}\left(1+3e^{-0.9t}\right)+270e^{-0.1t}e^{-0.9t}}{\left(1+3e^{-0.9t}\right)^2}$ oe $\frac{-10e^{-0.1t}+240e^{-1t}}{\left(1+3e^{-0.9t}\right)^2}$ simplified

Eg using product rule $\left(\frac{dP}{dt}\right) = -10e^{-0.1t}\left(1+3e^{-0.9t}\right)^{-1} + 270e^{-0.1t}e^{-0.9t}\left(1+3e^{-0.9t}\right)^{-2}$ oe

Remember to isw after a correct (unsimplified) answer.

There is no need to have the $\frac{dP}{dt}$ and it could be called $\frac{dy}{dx}$

(c)(i) Do NOT allow any marks in here without sight/implication of $\frac{dP}{dt}=0, \frac{dP}{dt}<0$ OR $\frac{dP}{dt}>0$

The question requires the candidate to find $t$ using part (b) so it is possible to do this part using inequalities using the same criteria as we apply for the equality. All marks in (c) can be scored from an incorrect denominator (most likely $v$ ), no denominator, or using a numerator the wrong way around ie $uv'-u'v$

M1 Sets their $\frac{dP}{dt}=0$ or the numerator of their $\frac{dP}{dt}=0$, factorises out or cancels a term in $e^{-0.1t}$ to reach a form $Ae^{\pm 0.9t}=B$ oe. Alternatively they could combine terms to reach $Ae^{-t}=Be^{-0.1t}$ or equivalent

Condone a double error on $e^{-0.1t}\times e^{-0.9t}=e^{-0.1t\times -0.9t}$ or similar before factorising. **Look for correct indices**.

If they use the product rule then expect to see their $\frac{dP}{dt}=0$ followed by multiplication of $\left(1+3e^{-0.9t}\right)^2$ before similar work to the quotient rule leads to a form $Ae^{\pm 0.9t}=B$

M1 Having set the numerator of their $\frac{dP}{dt}=0$ and obtained either $e^{\pm kt}=C$ ($k$ may be incorrect) or $Ae^{-t}=Be^{-0.1t}$ it is awarded for the correct order of operations, taking ln's leading to $t=..$

It cannot be awarded from impossible equations Eg $e^{\pm 0.9t}=-0.3$

A1 cso $t=\text{awrt } 3.53$ Accept $t=\frac{10}{9}\ln(24)$ or exact equivalent.

(c)(ii)

A1 awrt 102 following 3.53 The M's must have been awarded. This is not a B mark.

(d)

B1 Sight of 40

Condone statements such as $P\to 40$ $k\ldots 40$ or likewise