MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Exponentials and Logarithms - C3 (Pearson Edexcel)

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3. The value of a car is modelled by the formula

$$V = 16\,000e^{-kt} + A, \qquad t \geqslant 0,\ t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.

Given that the value of the car is £17 500 when new and £13 500 two years later,

(a) find the value of A,

(1)

(b) show that $k = \ln\left(\frac{2}{\sqrt{3}}\right)$

(4)

(c) Find the age of the car, in years, when the value of the car is £6000

Give your answer to 2 decimal places.

(4)

Question Number	Scheme	Marks
3(a)	$A = 1500$	B1
		(1)
(b)	Sub $t = 2, V = 13500 \Rightarrow 16000\mathrm{e}^{-2k} = 12000$	M1
	$\Rightarrow e^{-2k} = \frac{3}{4} \quad 0.75$ oe	A1
	$\Rightarrow k = -\frac{1}{2}\ln\frac{3}{4}, = \ln\sqrt{\frac{4}{3}} = \ln\left(\frac{2}{\sqrt{3}}\right)$	dM1, A1*
		(4)
(c)	Sub $6000 = 16000\mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} + '1500' \Rightarrow \mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = C$	M1
	$\Rightarrow \mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = \frac{45}{160} = 0.28125$	A1
	$\Rightarrow T = -\frac{\ln\left(\frac{45}{160}\right)}{\ln\left(\frac{2}{\sqrt{3}}\right)} = 8.82$	M1 A1
		(4)
		(9 marks)
Alt (b)	Sub $t = 2, V = 13500 \Rightarrow 13500 = 16000\mathrm{e}^{-2k} + '1500' \Rightarrow 1600e^{-2k} = 1200$	M1
	$\Rightarrow \ln 1600 - 2k = \ln 1200$	A1
	$\Rightarrow k = -\frac{1}{2}\ln\frac{1200}{1600}, = \ln\sqrt{\frac{4}{3}} = \ln\left(\frac{2}{\sqrt{3}}\right)$	dM A1*
		(4)

You may mark parts (a) and (b) together

(a)

B1: Sight of $A = 1500$

(b)

M1: Substitutes $t = 2, V = 13500 \Rightarrow 13500 = 16000\mathrm{e}^{-2k} + '\textit{their}\ 1500'$ and proceeds to $P\mathrm{e}^{-2k} = ...$ or $Q\mathrm{e}^{2k} = ...$ Condone slips, for example, V may be 1350. It is for an **attempt** to make $\mathrm{e}^{\pm 2k}$ the subject.

A1: $e^{-2k} = \frac{3}{4} \quad 0.75 \quad$ or $e^{2k} = \frac{4}{3} \quad \left(1.\dot{3}\right)$ oe

dM1: For taking ln's and proceeding to $k = ...$ For example $k = -\frac{1}{2}\ln\frac{3}{4}$ oe

May be implied by the correct decimal answer awrt 0.144 . This mark cannot be awarded from impossible to solve equations, that is ones of the type $\Rightarrow e^{\pm 2k} = c, \quad c \leqslant 0$

A1*: cso $k=\ln\left(\frac{2}{\sqrt{3}}\right)$ (brackets not required) **with a correct intermediate line** of either

$$\frac{1}{2}\ln\frac{4}{3},\ \frac{1}{2}\ln 4-\frac{1}{2}\ln 3,\ \ln\sqrt{\frac{4}{3}}\ \text{or}\ \ln\left(\frac{3}{4}\right)^{-\frac{1}{2}}$$

Note: $e^{-2k}=\frac{3}{4}\Rightarrow e^{2k}=\frac{4}{3}\Rightarrow e^{k}=\frac{2}{\sqrt{3}}$ are perfectly acceptable steps

See scheme for alternative method when ln's are taken before e^{-2k} is made the subject.

It is also possible to substitute $k=\ln\left(\frac{2}{\sqrt{3}}\right)$ into $13500=16000e^{-k\times 2}+1500$ and show that 12000 =12000 or similar. This is fine as long as a minimal conclusion (eg ✓)is given for the A1*.

(c)

M1: Sub $V=6000\Rightarrow 6000=16000e^{\pm kT}+\text{'their 1500'}$ and proceeds to $e^{\pm kT}=c,\quad c>0$

Allow candidates to write $k=$ awrt 0.144 or leave as '*k*'. Condone slips on *k*. Eg $k=2\ln\left(\frac{2}{\sqrt{3}}\right)$

Allow this when the = sign is replaced by any inequality.

If the candidate attempts to simplify the exponential function score for $\left(\frac{2}{\sqrt{3}}\right)^{\pm T}=c,\quad c>0$

A1: $e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T}=\frac{45}{160}=0.28125$, $e^{-kT}=\frac{45}{160}$ or $\left(\frac{2}{\sqrt{3}}\right)^{-T}=\frac{45}{160}$ Condone inequalities for =

Allow solutions from rounded values (3sf). Eg. $e^{-0.144T}=0.281$

M1: Correct order of operations using ln's and division leading to a value of T . It is implied by awrt 8.8

$\left(\frac{2}{\sqrt{3}}\right)^{-T}=\frac{45}{160}\Rightarrow -T=\log_{\frac{2}{\sqrt{3}}}\frac{45}{160}$ is equivalent work for this M mark.

A1: cso 8.82 only following correct work. Note that this is not awrt
Allow a solution using an inequality as long as it arrives at the solution 8.82.

....

There may be solutions using trial and improvement. Score (in this order) as follows

M1: Trial at value of $V=16000e^{-0.144t}+1500$ (oe) at either t = 8 or t = 9 and shows evidence
$V_{t=8}=awrt\ 6500$ $V_{t=9}=awrt\ 5900$ This may be implied by the subsequent M1

M1: Trial at value of $V=16000e^{-0.144t}+1500$ (oe) at either t = 8.81 or t = 8.82 and shows evidence.
(See below for answers. Allow to 2sf)

A1: Correct answers for *V* at **both** *t* = 8.81 **and** *t* = 8.82 $V_{t=8.81}=awrt\ 6006$ $V_{t=8.82}=awrt\ 5999$

A1: Correctly deduces 8.82 with all evidence.
Hence candidates who **just** write down 8.82 will score 1, 1, 0, 0

....

Leave blank

2. Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y+5) = 2 + \ln(4-y)$

(4)

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Question Number	Scheme	Marks
2.(a)	$e^{3x-9}=8 \Rightarrow 3x-9=\ln 8$	M1
	$\Rightarrow x=\dfrac{\ln 8+9}{3}, =\ln 2+3$	A1, A1
		(3)
(b)	$\ln(2y+5)=2+\ln(4-y)$	
	$\ln\left(\dfrac{2y+5}{4-y}\right)=2$	M1
	$\left(\dfrac{2y+5}{4-y}\right)=e^2$	M1
	$2y+5=e^2(4-y) \Rightarrow 2y+e^2y=4e^2-5 \Rightarrow y=\dfrac{4e^2-5}{2+e^2}$	dM1, A1
		(4)
		7 marks

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\dfrac{\ln 8+9}{3}$ or equivalent such as $\dfrac{\ln 8e^9}{3}$, $3+\ln\left(\sqrt[3]{8}\right)$, $\dfrac{\log 8}{3\log e}+3$ or even 3.69

A1 cso $\ln 2+3$. Accept $\ln 2e^3$

....

Alt I (a)

$e^{3x-9}=8 \Rightarrow \dfrac{e^{3x}}{e^9}=8 \Rightarrow e^{3x}=8e^9 \Rightarrow 3x=\ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3}=\sqrt[3]{8} \Rightarrow x-3=\ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9}=2 \Rightarrow x-9=\ln 2$)

....

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2+\ln(4-y)=\ln\left(e^2(4-y)\right)$ or $\ln(2y+5)-\ln(4-y)=\ln\left(\dfrac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the e^2

M1 Scored for an attempt to undo the ln's to get an equation in *y* This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y))=2 \Rightarrow g(y)=e^2$ and can be scored eg where $g(y)=2y+5-(4-y)$

It cannot be awarded for just $2y+5=e^2+4-y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making *y* the subject. Expect to see both terms in *y* collected and factorised (may be implied) before reaching *y* =. Condone slips, for eg, on signs. *y* =2.615 scores this.

A1 $y=\dfrac{4e^2-5}{2+e^2}$ or equivalent such as $y=4-\dfrac{13}{2+e^2}$ ISW after you see the correct answer.

....

Special Case: $\ln(2y+5)-\ln(4-y)=2 \Rightarrow \dfrac{\ln(2y+5)}{\ln(4-y)}=2 \Rightarrow \dfrac{2y+5}{4-y}=e^2 \Rightarrow$ Correct answer score M0 M1 M1 A0

Leave blank

9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a\ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

Question	Scheme	Marks
9(a)	Subs $D=15$ and $t=4$ $\quad x=15e^{-0.2\times4}=6.740\ (mg)$	M1A1
		(2)
(b)	$15e^{-0.2\times7}+15e^{-0.2\times2}=13.754(mg)$	M1A1*
		(2)
(c)	$15e^{-0.2\times T}+15e^{-0.2\times(T+5)}=7.5$	M1
	$15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$	
	$15e^{-0.2\times T}(1+e^{-1})=7.5\Rightarrow e^{-0.2\times T}=\dfrac{7.5}{15(1+e^{-1})}$	dM1
	$T=-5\ln\left(\dfrac{7.5}{15(1+e^{-1})}\right)=5\ln\left(2+\dfrac{2}{e}\right)$	A1, A1
		(4)
		(8 marks)

(a)

M1 Attempts to substitute both $D=15$ and $t=4$ in $x=De^{-0.2t}$

It can be implied by sight of $15e^{-0.8}$, $15e^{-0.2\times4}$ or awrt 6.7

Condone slips on the power. Eg you may see -0.02

A1 CAO 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1 Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7

Evidence would be $15e^{-0.2\times7}+15e^{-0.2\times2}$ or similar expressions such as $\left(15e^{-1}+15\right)e^{-0.2\times2}$

Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**

Alternatively finds the amount after 5 hours, $15e^{-1}=$ awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $e^{-0.4}$ to get awrt **13.75**.

Sight of 5.52+15=20.52$\rightarrow$ 13.75 is fine.

A1* cso so both the expression $15e^{-0.2\times7}+15e^{-0.2\times2}$ and $13.754(mg)$ are required

Alternatively both the expression $\left(15e^{-0.2\times5}+15\right)\times e^{-0.2\times2}$ and $13.754(mg)$ are required.

Sight of just the numbers is not enough for the A1*

(c)

M1 Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing

Eg. accept $15e^{-0.2\times T}+15e^{-0.2\times(T\pm5)}=7.5$ or similar equations $\left(15e^{-1}+15\right)e^{-0.2\times T}=7.5$

dM1 Attempts to solve their equation, dependent upon the previous mark, by proceeding to $e^{-0.2\times T}=\ldots$

An attempt should involve an attempt at the index law $x^{m+n}=x^m\times x^n$ and taking out a factor of $e^{-0.2\times T}$ Also score for candidates who make $e^{+0.2\times T}$ the subject using the same criteria

A1 Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15(1+e^{-1})}\right)$

A1 CSO $T=5\ln\left(2+\dfrac{2}{e}\right)$ Condone t appearing for T throughout this question.

Alt (c) using lns

(c)	$15e^{-0.2\times T}+15e^{-0.2\times(T+5)}=7.5$ $15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$ $e^{-0.2\times T}(1+e^{-1})=0.5\Rightarrow -0.2\times T+\ln(1+e^{-1})=\ln 0.5$ $\Rightarrow T=\dfrac{\ln 0.5-\ln(1+e^{-1})}{-0.2},\Rightarrow T=5\ln\left(2+\dfrac{2}{e}\right)$	M1 dM1 A1, A1 **(4)** **(8 marks)**

You may see numerical attempts at part (c).

Such an attempt can score a maximum of two marks.

This can be achieved either by

Method One

1st Mark (Method): $15e^{-0.2\times T}+\text{awrt }5.52e^{-0.2\times T}=7.5\Rightarrow e^{-0.2\times T}=\text{awrt }0.37$

2nd Mark (Accuracy): $\text{T=-5ln}(\text{awrt }0.37)$ or awrt 5.03 or $\text{T=-5ln}\left(\dfrac{7.5}{\text{awrt }20.52}\right)$

Method Two

1st Mark (Method): $13.754e^{-0.2\times T}=7.5\Rightarrow T=-5\ln\left(\dfrac{7.5}{13.754}\right)$ or equivalent such as 3.03

2nd Mark (Accuracy): 3.03+2=5.03 Allow $-5\ln\left(\dfrac{7.5}{13.754}\right)+2$

Method Three (by trial and improvement)

1st Mark (Method): $15e^{-0.2\times 5}+15e^{-0.2\times 10}=7.55$ or $15e^{-0.2\times 5.1}+15e^{-0.2\times 10.1}=7.40$ or any value between

2nd Mark (Accuracy): Answer T=5.03.

Leave blank

4. Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \qquad 0 \leqslant t \leqslant T$$

(a) State the value of θ when $t = 0$

(1)

Given that the temperature of the water in the kettle is 70 °C when $t = 40$,

(b) find the exact value of λ, giving your answer in the form $\dfrac{\ln a}{b}$, where a and b are integers.

(4)

When $t = T$, the temperature of the water reaches 100 °C and the kettle switches off.

(c) Calculate the value of T to the nearest whole number.

(2)

Question Number	Scheme	Marks
4(a)	$(\theta =)20$	B1 **(1)**
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \frac{\ln 2}{40}$	M1A1
		(4)
(c)	$\theta = 100 \Rightarrow T = \frac{\ln 0.2}{-\text{their}'\lambda'}$	M1
	$T = \text{awrt } 93$	A1
		(2)
		(7 marks)
Alt (b)	Sub $t = 40, \theta = 70 \Rightarrow 100e^{-40\lambda} = 50$	
	$\Rightarrow \ln 100 - 40\lambda = \ln 50$	M1A1
	$\Rightarrow \lambda = \frac{\ln 100 - \ln 50}{40} = \frac{\ln 2}{40}$	M1A1
		(4)

(a)

B1 Sight of $(\theta =)20$

(b)

M1 Sub $t=40, \theta=70 \Rightarrow 70=120-100e^{-40\lambda}$ and proceed to $e^{\pm 40\lambda}=A$ where A is a constant. Allow sign slips and copying errors.

A1 $e^{-40\lambda}=0.5$ or $e^{40\lambda}=2$ or exact equivalent

M1 For undoing the e's by taking ln's and proceeding to $\lambda = ..$

May be implied by the correct decimal answer awrt 0.017 or $\lambda = \frac{\ln 0.5}{-40}$

A1 cso $\lambda = \frac{\ln 2}{40}$

Accept equivalents in the form $\frac{\ln a}{b}$, $a, b \in \mathbb{Z}$ such as $\lambda = \frac{\ln 4}{80}$

(c)

M1 Substitutes $\theta = 100$ and their numerical value of λ into $\theta = 120 - 100e^{-\lambda t}$ and proceed to $T = \pm \frac{\ln 0.2}{\text{their '}\lambda\text{'}}$ or $T = \pm \frac{\ln 5}{\text{their '}\lambda\text{'}}$ Allow inequalities here.

A1 awrt $T = 93$

Watch for candidates who lose the minus sign in (b) and use $\lambda = \frac{\ln \frac{1}{2}}{40}$ in (c). Many then reach $T = -93$ and ignore the minus. This is M1 A0

Leave blank

8. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geqslant 0, \quad t \in \mathbb{R}$$

(a) Calculate the number of primroses at the start of the study.
(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.
(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.
(4)

(d) Explain why the population of primroses can never be 270
(1)

Question Number	Scheme	Marks
8.(a)	$P=\frac{800e^0}{1+3e^0},=\frac{800}{1+3}=200$	M1,A1
		(2)
(b)	$250=\frac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250,\ \Rightarrow e^{0.1t}=5$	M1,A1
	$t=\frac{1}{0.1}\ln(5)$	M1
	$t=10\ln(5)$	A1
		(4)
(c)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}\Rightarrow\frac{dP}{dt}=\frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t}-800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$	M1,A1
	At t=10	
	$\frac{dP}{dt}=\frac{(1+3e)\times 80e-240e^2}{(1+3e)^2}=\frac{80e}{(1+3e)^2}$	M1,A1
		(4)
(d)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}=\frac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\frac{800}{3}=266$. Hence P cannot be 270	B1
		(1)
		(11 marks)

(a)

M1 Sub t = 0 into P **and** use $e^0=1$ in at least one of the two cases. Accept $P=\frac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P=\frac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t}=B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t}=D$

A1 $e^{0.1t}=5$ or $e^{-0.1t}=0.2$

M1 Dependent upon gaining $e^{0.1t} = E$, for taking ln's of both sides and proceeding to t=…

Accept $e^{0.1t} = E \Rightarrow 0.1t = \ln E \Rightarrow t = ...$ It could be implied by $t = \text{awrt } 16.1$

A1 $t = 10\ln(5)$

Accept exact equivalents of this as long as a and b are integers. Eg. $t = 5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$ and NOT $kte^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u = 800e^{0.1t}, v = 1+3e^{0.1t}, u' = pe^{0.1t}, v' = qe^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form

$$\frac{(1+3e^{0.1t})\times pe^{0.1t} - 800e^{0.1t}\times qe^{0.1t}}{(1+3e^{0.1t})^2}$$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$

For the chain rule look for

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow \frac{dP}{dt} = 800\times\left(e^{-0.1t}+3\right)^{-2}\times -0.1e^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$$

M1 For substituting $t = 10$ into their $\frac{dP}{dt}$, NOT P

Accept numerical answers for this. 2.59 is the numerical value if $\frac{dP}{dt}$ was correct

A1 $\frac{dP}{dt} = \frac{80e}{(1+3e)^2}$ or equivalent such as $\frac{dP}{dt} = 80e(1+3e)^{-2}, \frac{80e}{1+6e+9e^2}$

Note that candidates who substitute $t = 10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow 0.1t = \ln(-27)$ which has no answers.

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow e^{0.1t} / e^x$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of t ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment