MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Trigonometry - C3 (Pearson Edexcel)

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6. (i) Using the identity for $\tan(A \pm B)$, solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (a) Using the identity for $\tan(A \pm B)$, show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°,\ n \in \mathbb{Z}$$

(2)

(b) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(5)

Question Number	Scheme	Marks
6(i)	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan(2x+32°)=5$	B1
	$\Rightarrow x=\dfrac{\arctan 5-32°}{2}$	M1
	$\Rightarrow x=\text{awrt } 23.35°, -66.65°$	A1A1
		(4)
(ii)(a)	$\tan(3\theta-45°)=\dfrac{\tan 3\theta-\tan 45°}{1+\tan 45°\tan 3\theta}=\dfrac{\tan 3\theta-1}{1+\tan 3\theta}$	M1A1*
		(2)
(b)	$(1+\tan 3\theta)\tan(\theta+28°)=\tan 3\theta-1$	
	$\Rightarrow \tan(\theta+28°)=\tan(3\theta-45°)$	B1
	$\theta+28°=3\theta-45°\Rightarrow\theta=36.5°$	M1A1
	$\theta+28°+180°=3\theta-45°\Rightarrow\theta=126.5°$	dM1A1
		(5)
		(11 marks)
6(i) ALT 1	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan 2x=\dfrac{5-\tan 32°}{1+5\tan 32°}=awrt 1.06$	B1
	$\Rightarrow x=\dfrac{\arctan\left(\dfrac{5-\tan 32°}{1+5\tan 32°}\right)}{2}$	M1
	$\Rightarrow x=23.35°, -66.65°$	A1A1
		(4)
6(ii) ALT 2	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow\dfrac{2\tan x}{1-\tan^2 x}+\tan 32°=5-5\times\dfrac{2\tan x}{1-\tan^2 x}\tan 32°$	
	$\Rightarrow(5-\tan 32°)\tan^2 x+(2+10\tan 32°)\tan x+\tan 32°-5=0$	
	OR $\Rightarrow awrt\ 4.38\tan^2 x+8.25\tan x-4.38=0$	B1
	Quadratic formula $\Rightarrow\tan x=0.4316,-2.3169\Rightarrow x=..$	M1
	$\Rightarrow x=23.35°, -66.65°$	A1 A1
		(4)

(i)

B1: Stating or implying by subsequent work $\tan(2x+32°)=5$

M1: Scored for the correct order of operations from $\tan(2x\pm 32°)=5$ to $x = ..$ $x=\dfrac{\arctan 5\pm 32°}{2}$

This may be implied by one correct answer

A1: One of awrt $x=23.3/23.4°, -66.6/-66.7°$ **One dp accuracy required for this penultimate mark.**

A1: Both of $x=\text{awrt } 23.35°, -66.65°$ and no other solutions in the range $-90° < x < 90°$

Using Alt I
B1: $\tan 2x = \text{awrt}1.06$
M1: For attempting to make $\tan 2x$ the subject followed by correct inverse operations to find a value for x from their $\tan 2x = k$
If they write down $\tan(2x+32°) = 5$ and then the answers that is fine for all 4 marks.
Answers mixing degrees and radians can only score the first B1

(ii)(a)

M1: States or implies (just rhs) $\tan(3\theta - 45°) = \dfrac{\tan 3\theta \pm \tan 45°}{1 \pm \tan 45° \tan 3\theta}$

A1*: Complete proof with the lhs, the correct identity $\dfrac{\tan 3\theta - \tan 45°}{1 + \tan 45° \tan 3\theta}$ and either stating that $\tan 45° = 1$ or substituting $\tan 45° = 1$ (which may only be seen on the numerator) and proceeding to given answer.
It is possible to work backwards here $\dfrac{\tan 3\theta - 1}{1 + \tan 3\theta} = \dfrac{\tan 3\theta - \tan 45°}{1 + \tan 45° \tan 3\theta} = \tan(3\theta - 45°)$ with M1 A1 scored at the end. Do not allow the final A1* if there are errors.

(ii)(b)

B1: Uses (ii)(a) to state or imply that $\tan(\theta + 28°) = \tan(3\theta - 45°)$
Allow this mark for $(1 + \tan 3\theta)\tan(\theta + 28°) = (1 + \tan 3\theta)\tan(3\theta - 45°)$

M1: $\theta + 28° = 3\theta - 45° \Rightarrow \theta = ..$
We have seen two incorrect methods that should be given M0.
$\tan(\theta + 28°) = \tan(3\theta - 45°) \Rightarrow \tan(3\theta - 45°) - \tan(\theta + 28°) = 0 \Rightarrow (3\theta - 45°) - (\theta + 28°) = 0 \Rightarrow \theta = ...$
and $\tan 3\theta - \tan 45° = \tan\theta + \tan 28° \Rightarrow 3\theta - 45° = \theta + 28° \Rightarrow \theta = ...$

A1: $\theta = 36.5°$ oe such as $\dfrac{73}{2}$

dM1: A correct method of finding a 2nd solution $\theta + 28° + 180° = 3\theta - 45° \Rightarrow \theta = ..$ The previous M must have been awarded. The method may be implied by their $\theta_1 + 90°$ but only if the previous M was scored.
It is an incorrect method to substitute the acute angle into one side of $\tan(\theta + 28°) = \tan(3\theta - 45°)$
Eg. $\tan(36.5 + 28°) = \tan(3\theta - 45°)$ and use trig to find another solution.

A1: $\theta = 36.5°, 126.5°$ oe and no other solutions in the range.

The questions states 'hence' so the minimum expected working is $\tan(\theta + 28°) = \tan(3\theta - 45°)$. Full marks can be awarded when this point is reached.
(ii) (b) Alternative solution using compound angles.
……..
(ii) (b) Alternative solution using compound angles.
From the B1 mark, $\tan(\theta + 28°) = \tan(3\theta - 45°)$ they proceed to

$$\frac{\sin(\theta + 28°)}{\cos(\theta + 28°)} = \frac{\sin(3\theta - 45°)}{\cos(3\theta - 45°)} \Rightarrow \sin((3\theta - 45°) - (\theta + 28°)) = 0$$ via the compound angle identity

So, M1 is gained for an attempt at one value for $\sin(2\theta - 73°) = 0$, condoning slips andA1 for $\theta = 36.5°$
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9. (a) Express $\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 decimal places.

(3)

$$M(\theta) = 40 + (3\sin\theta - 6\cos\theta)^2$$

(b) Find

(i) the maximum value of $M(\theta)$,

(ii) the smallest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $M(\theta)$ occurs.

(3)

$$N(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2\cos 2\theta)^2}$$

(c) Find

(i) the maximum value of $N(\theta)$,

(ii) the largest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $N(\theta)$ occurs.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Question Number	Scheme	Marks
9.(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=2\Rightarrow\alpha=\text{awrt }1.107$	M1A1
		(3)
(b)(i)	$'40+9R^2'=85$	M1A1
(ii)	$\theta=\frac{\pi}{2}+1.107\Rightarrow\theta=\text{awrt }2.68$	B1ft
		(3)
(c)(i)	6	B1
(ii)	$2\theta-'1.107'=3\pi\Rightarrow\theta=\text{awrt }5.27$	M1A1
		(3)
		(9 marks)

(a)

B1: Accept $R=\sqrt{5}$ **Do not accept** $R=\pm\sqrt{5}$

M1: For sight of $\tan\alpha=\pm 2,\ \tan\alpha=\pm\frac{1}{2}$. Condone $\sin\alpha=2,\cos\alpha=1\Rightarrow\tan\alpha=\frac{2}{1}$

If R is found first, accept $\sin\alpha=\pm\frac{2}{R},\ \cos\alpha=\pm\frac{1}{R}$

A1: $\alpha=\text{awrt }1.107$. The degrees equivalent $63.4°$ is A0.

(b)(i)

M1: Attempts $'40+9R^2'$ OR $'40+3R^2'$ using their R.

Can be scored for sight of the statement $'40+9R^2'$

It can be done via calculus. The M mark will probably be awarded when $\left("\alpha"-\frac{\pi}{2}\right)=-0.464$ is substituted into $M(\theta)$

A1: 85 exactly. Without any method this scores both marks. Do not accept awrt 85.

(b)(ii)

B1ft: For awrt 2.68 or $\left(\frac{\pi}{2}+"\alpha"\right)$ A simple way would be to add 1.57 to their α to 2dp

Accept awrt $153.4°$ for candidates who work in degrees. Follow through in degrees on $90°+'\alpha'$

(c)(i)

B1: 6

(c)(ii)

M1: Using $2\theta\pm'1.107'=n\pi$ where n is a positive integer leading to a value for θ

In degrees for $2\theta\pm\text{their}'63.43'=180n$ where n is a positive integer leading to a value for θ

Another alternative is to solve $\tan 2\theta=2$ so score for $\frac{180n+\arctan 2}{2}$ or $\frac{\pi n+\arctan 2}{2}$

A1: $\theta=\text{awrt }5.27$ or if candidate works in degrees awrt $301.7°$

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4. (a) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 \leqslant \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question Number	Scheme	Marks
4.(a)	$R=\sqrt{29}$	B1
	$\tan\alpha=\frac{2}{5}\Rightarrow\alpha=\text{awrt } 0.381$	M1A1
		(3)
(b)	$5\cot 2x-3\operatorname{cosec}2x=2\Rightarrow 5\frac{\cos 2x}{\sin 2x}-\frac{3}{\sin 2x}=2$	M1
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1
		(2)
(c)	$5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\frac{3}{\sqrt{29}}$	M1
	$2x+0.381=\arccos\left(\frac{3}{\sqrt{29}}\right)\Rightarrow x=...$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)
		(9 marks)
Alt I (c)	$5\cos 2x-2\sin 2x=3\Rightarrow 10\cos^2 x-5-4\sin x\cos x=3$	
	$\Rightarrow 4\tan^2 x+2\tan x-1=0$	M1
	$\Rightarrow\tan x=\frac{-1\pm\sqrt{5}}{4}\Rightarrow x=..$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)
Alt II (c)	$5\cos 2x-2\sin 2x=3\Rightarrow(5\cos 2x)^2=(3+2\sin 2x)^2$ **&** $\cos^2 2x=1-\sin^2 2x$	
	$\Rightarrow 29\sin^2 2x+12\sin 2x-16=0$	M1
	$\Rightarrow\sin 2x=\frac{-12\pm\sqrt{2000}}{58}\Rightarrow 2x=..\Rightarrow x=..$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)

(a)

B1 $R=\sqrt{29}$

Condone $R=\pm\sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$)

M1 $\tan\alpha=\pm\frac{2}{5},\ \tan\alpha=\pm\frac{5}{2}\Rightarrow\alpha=...$

If R is used to find α accept $\sin\alpha=\pm\frac{2}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=...$

A1 $\alpha = \text{awrt } 0.381$ Note that the degree equivalent $\alpha = \text{awrt } 21.8°$ is A0

(b)

M1 Replaces $\cot 2x$ by $\frac{\cos 2x}{\sin 2x}$ **and** $\operatorname{cosec} 2x$ by $\frac{1}{\sin 2x}$ in the lhs
Do not be concerned by the coefficients 5 and -3.
Replacing $\cot 2x$ by $\frac{1}{\tan 2x}$ does not score marks until the $\tan 2x$ has been replaced by $\frac{\sin 2x}{\cos 2x}$
They may state $\times \sin 2x \Rightarrow 5\cos 2x - 3 = 2\sin 2x$ which implies this mark

A1 cso $5\cos 2x - 2\sin 2x = 3$ There is no need to state the value of 'c'
The notation must be correct. They cannot mix variables within their equation
Do not accept for the final A1 $\tan 2x = \frac{\sin}{\cos} 2x$ within their equations

(c)

M1 Attempts to use part (a) and (b). They must be using their R and α from part (a) and their c from part (b)
Accept $\cos(2x \pm '\alpha') = \frac{'c'}{'R'}$ Condone $\cos(\theta \pm '\alpha') = \frac{'c'}{'R'}$ or even $\cos(x \pm '\alpha') = \frac{'c'}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$
Don't be concerned if they change the variable in the question and solve for $\theta =$ (as long as all operations have been undone).You may not see any working. It is implied by one correct answer.
You may need to check with a calculator.
Eg for an incorrect α $\cos(2x + 1.19) = \frac{3}{\sqrt{29}} \Rightarrow x = -0.105$ would score M1 dM1 A0 A0

A1 One solution correct, usually $x = 0.3 / 0.30$ or $x = 2.46$ or in degrees $17.2°$ **or** $141.(0)°$

A1 Both solutions correct awrt $x = \text{awrt } 0.30, 2.46$ and no extra values in the range.
Condone candidates who write 0.3 and 2.46 without any (more accurate) answers
In degrees accept awrt 1 dp $17.2°, 141.(0)°$ and no extra values in the range.

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Special case: For candidates who are misreading the question and using their part (a) with 2 on the rhs.
They will be allowed to score a maximum of SC M1 dM1 A0 A0

M1 Attempts to use part (a) with 2. They must be using their R and α from part (a)
Accept $\cos(2x \pm '\alpha') = \frac{2}{'R'}$ Condone $\cos(\theta \pm '\alpha') = \frac{2}{'R'}$ or even $\cos(x \pm '\alpha') = \frac{2}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$
You may not see any working. It is implied by one correct answer. You may need to check with a calculator.
Eg for an correct α and R $\cos(2x + 0.381) = \frac{2}{\sqrt{29}} \Rightarrow x = 0.405$

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Alt to part (c)

M1 Attempts both double angle formulae condoning sign slips on cos2x , divides by $\cos^2 x$ and forms a quadratic in tan by using the identity $\pm 1 \pm \tan^2 x = \sec^2 x$

dM1 Attempts to solve their quadratic in tan$x$ leading to a solution for x.

A1 A1 As above

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9. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \qquad n \in \mathbb{Z}$$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3\tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question Number	Scheme	Marks
9(a)	$\sin 2x - \tan x = 2\sin x\cos x - \tan x$	M1
	$= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\sin x}{\cos x} \times (2\cos^2 x - 1)$	
	$= \tan x \cos 2x$	dM1 A1*
		(4)
(b)	$\tan x\cos 2x = 3\tan x\sin x \Rightarrow \tan x(\cos 2x - 3\sin x) = 0$	
	$\cos 2x - 3\sin x = 0$	M1
	$\Rightarrow 1 - 2\sin^2 x - 3\sin x = 0$	M1
	$\Rightarrow 2\sin^2 x + 3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-3 \pm \sqrt{17}}{4} \Rightarrow x = ...$	M1
	Two of $x = 16.3°, 163.7°, 0, 180°$	A1
	All four of $x = 16.3°, 163.7°, 0, 180°$	A1
		(5)
		(9 marks)

(a)

M1 Uses a correct double angle identity involving $\sin 2x$ Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\sin 2x = 2\sin x\cos x$ and attempts to combine the two terms using a common denominator. This can be awarded on two separate terms with a common denominator.
Alternatively uses $\sin x = \tan x\cos x$ and attempts to combine two terms using factorisation of tan x

dM1 Both M's must have been scored. Uses a correct double angle identity involving $\cos 2x$.

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent
Withhold this mark if for instance they write $\tan x = \dfrac{\sin}{\cos}$

If the candidate $\times \cos x$ on line 1 and/or $\div \sin x$ they cannot score any more than one mark unless they are working with both sides of the equation or it is fully explained.

(b)

M1 The $\tan x$ must be cancelled or factorised out to produce $\cos 2x - 3\sin x = 0$ or $\dfrac{\cos 2x}{\sin x} = 3$ oe Condone slips

M1 Uses $\cos 2x = 1 - 2\sin^2 x$ to form a 3TQ=0 in $\sin x$ The = 0 may be implied by later work

M1 Uses the formula/completion of square or GC with invsin to produce at least one value for x
It may be implied by one correct value.
This mark **can** be scored from factorisation of their 3TQ in $\sin x$ **but only if** their quadratic factorises.

A1 Two of $x = 0, 180°, \text{awrt } 16.3°, \text{awrt } 163.7°$ or in radians two of awrt 0.28, 2.86, 0 and π or 3.14
This mark can be awarded as a SC for those students who just produce $0, 180°$ (or 0 and π) from $\tan x = 0$ or $\sin x = 0$.

A1 All four values in degrees $x = 0, 180°, \text{awrt } 16.3°, \text{awrt } 163.7°$ and no extra's inside the range $0 \leqslant x < 360°$.
Condone $0 = 0.0$ and $180° = 180.0°$ Ignore any roots outside range.

Alternatives to parts (a) and (b)

(a) Alt 1	$\tan x\cos 2x = \tan x\left(2\cos^2 x - 1\right)$	M1
	$= 2\tan x\cos^2 x - \tan x$	
	$= 2\dfrac{\sin x}{\cos x}\cos^2 x - \tan x$	M1
	$= 2\sin x\cos x - \tan x$	
	$= \sin 2x - \tan x$	dM1 A1
		(4)

a) Alt 1 Starting from the rhs

M1 Uses a correct double angle identity for $\cos 2x$. Accept any correct version including $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and attempts to multiply out the bracket

dM1 Both M's must have been scored. It is for using $2\sin x\cos x =$ $\sin 2x$

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent. See Main scheme for how to deal with candidates who $\div \tan x$

(a) Alt 2	$\sin 2x - \tan x \equiv \tan x\cos 2x$	
	$2\sin x\cos x - \tan x \equiv \tan x(2\cos^2 x - 1)$	M1
	$2\sin x\cos x - \cancel{\tan x} \equiv 2\tan x\cos^2 x - \cancel{\tan x}$	
	$2\sin x\cos x \equiv 2\dfrac{\sin x}{\cos x}\cos^2 x$	M1
	$2\sin x\cos x \equiv 2\sin x\cos x$	dM1
	+statement that it must be true	A1*

a) Alt 2 Candidates who use both sides

M1 Uses a correct double angle identity involving $\sin 2x$ **or** $\cos 2x$. Can be scored from either side
Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$ or $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and cancels the $\tan x$ term from both sides

dM1 Uses a correct double angle identity involving $\sin 2x$ Both previous M's must have been scored

A1* A fully correct solution with no errors or omissions AND statement "hence true", "a tick", "QED". W[5]
All notation must be correct and variables must be consistent

....

It is possible to solve part (b) without using the given identity. There are various ways of doing this, one of which is shown below.

$\sin 2x - \tan x = 3\tan x\sin x \Rightarrow \quad 2\sin x\cos x - \dfrac{\sin x}{\cos x} = 3\dfrac{\sin x}{\cos x}\sin x$

$2\sin x\cos^2 x - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ and $\cos x$

$2\sin x\left(1 - \sin^2 x\right) - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ only

$\left(2\sin^2 x + 3\sin x - 1\right)\sin x = 0$

$x = ..$ — M1 Solving equation to find at least one x

Two of $x = 16.3°, 163.7°, 0, 180°$ — A1

All four of $x = 16.3°, 163.7°, 0, 180°$ and no extras A1

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5. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x}\cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y, \quad 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p\,\text{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

Question	Scheme	Marks
5 (i)	$y = e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \frac{1}{4}\arctan\frac{3}{4}$	M1
	$\Rightarrow x = \text{awrt } 0.9463$ 4dp	A1 **(5)**
(ii)	$x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1 **(5)** **(10 marks)**
(ii) Alt I	$x = \sin^2 2y \Rightarrow x = \frac{1}{2} - \frac{1}{2}\cos 4y$	2nd M1
	$\frac{dx}{dy} = 2\sin 4y$	1st M1 A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1 **(5)**
(ii) Alt II	$x^{\frac{1}{2}} = \sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\frac{dy}{dx}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1 **(5)**
(ii) Alt III	$x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \text{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1 **(5)**

(i)

M1 Uses the product rule $uv' + vu'$ to achieve $\left(\frac{dy}{dx}\right) = Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A, B \neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx} = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx}=0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x}\equiv \tan 4x$, moves from $\tan 4x = C, C\neq 0$ using correct order of operations to $x=...$ Accept $x=$ awrt 0.16 (radians) $x=$ awrt 9.22 (degrees) for this mark.

If a candidate elects to pursue a more difficult method using $R\cos(\theta+\alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x-\text{awrt } 0.64)$ before using the correct order of operations to $x=...$

Similarly candidates who square $3\cos 4x-4\sin 4x=0$ then use a Pythagorean identity should proceed from either $\sin 4x=\frac{3}{5}$ or $\cos 4x=\frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x=$ awrt 0.9463.

Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.

There is no need for lhs to be seen/ correct

If the product rule is used look for $\cancel{\frac{dx}{dy}} = \pm A\sin 2y\cos 2y \pm B\sin 2y\cos 2y$,

A1 Both lhs and rhs correct (unsimplified) . $\frac{dx}{dy}=2\sin 2y\times 2\cos 2y=(4\sin 2y\cos 2y)$ or

$1=2\sin 2y\times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression.

You may just see a statement such as $4\sin 2y\cos 2y = 2\sin 4y$ which is fine.

Candidates who write $\cancel{\frac{dx}{dy}} = A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dx}{dy}} = \frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx} = 1/\frac{dx}{dy}$ **for their expression in *y***. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy}=2\sin 4y\Rightarrow\frac{dy}{dx}=2\text{cosec}4y$ is condoned for the M1

If $\frac{dx}{dy}=a+b$ do not allow $\frac{dy}{dx}=\frac{1}{a}+\frac{1}{b}$

A1 $\frac{dy}{dx}=\frac{1}{2}\text{cosec}4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x=\sin^2 2y\Rightarrow x=\pm\frac{1}{2}\pm\frac{1}{2}\cos 4y$ from $\cos 4y=\pm 1\pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}}=\sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}}=Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y=\text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\cancel{\frac{dy}{dx}}=N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}}\times x^{-0.5}$