# MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

## Chapters:

### Complex Numbers - FP1 (Pearson Edexcel)

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**9.** (i) Given that

$$\frac{3w+7}{5} = \frac{p-4\text{i}}{3-\text{i}} \qquad \text{where } p \text{ is a real constant}$$

(a) express $w$ in the form $a + b\text{i}$, where $a$ and $b$ are real constants. Give your answer in its simplest form in terms of $p$.

**(5)**

Given that $\arg w = -\frac{\pi}{2}$

(b) find the value of $p$.

**(1)**

(ii) Given that

$$(z + 1 - 2\text{i})^* = 4\text{i}z$$

find $z$, giving your answer in the form $z = x + \text{i}y$, where $x$ and $y$ are real constants.

**(6)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| **9.**(i) (a) | $\frac{3w+7}{5} = \frac{(p-4\text{i})}{(3-\text{i})} \times \frac{(3+\text{i})}{(3+\text{i})}$ | Multiplies by $\frac{(3+\text{i})}{(3+\text{i})}$ or divide by $(9-3\text{i})$ then multiply by $\frac{(9+3\text{i})}{(9+3\text{i})}$ | M1 |
| | $= \left(\frac{3p+4}{10}\right) + \left(\frac{p-12}{10}\right)\text{i}$ | Evidence of $(3-\text{i})(3+\text{i}) = 10$ or $3^2+1^2$ or $9^2+3^2$ | B1 |
| | So, $w = \left(\frac{3p-10}{6}\right) + \left(\frac{p-12}{6}\right)\text{i}$ | Rearranges to $w = ...$ | dM1 |
| | | At least one of either the real or imaginary part of $w$ is correct in any equivalent form. | A1 |
| | | Correct $w$ in the form $a + b\text{i}$. Accept $a + \text{i}b$. | A1 |
| | | | **[5]** |
| ALT (i) (a) | $(3-\text{i})(3w+7) = 5(p-4\text{i})$ | | |
| | $9w + 21 - 3\text{i}w - 7\text{i} = 5p - 20\text{i}$ | | |
| | $w(9-3\text{i}) = 5p - 21 - 13\text{i}$ | | |
| | Let $w = a + b\text{i}$, so $(a+b\text{i})(9-3\text{i}) = 5p - 21 - 13\text{i}$ | | |
| | $9a + 3b - 3a\text{i} + 9b\text{i} = 5p - 21 - 13\text{i}$ | | |
| | Real: $9a + 3b = 5p - 21$<br>Imaginary: $-3a + 9b = -13$ | Sets $w = a + b\text{i}$ and equates at least either the real or imaginary part. | M1 |
| | | $9a + 3b = 5p - 21$ | B1 |
| | $b = \frac{p-12}{6}$, $a = \frac{3p-10}{6}$ | Solves to finds $a = ...$ and $b = ...$ | dM1 |
| | | At least one of $a$ or $b$ is correct in any equivalent form. | A1 |
| | $w = \left(\frac{3p-10}{6}\right) + \left(\frac{p-12}{6}\right)\text{i}$ | Correct $w$ in the form $a + b\text{i}$. Accept $a + \text{i}b$. | A1 |
| | | | **[5]** |
| (b) | $\left\{\arg w = -\frac{\pi}{2} \Rightarrow \left(\frac{3p-10}{6}\right) = 0\right\} \Rightarrow p = \frac{10}{3}$ | $p = \frac{10}{3}$<br>Follow through provided $p < 12$ | B1ft |
| | | | **[1]** |

| | | | |
|---|---|---|---|
| (ii) | $(x+\mathrm{i}y+1-2\mathrm{i})^* = 4\mathrm{i}(x+\mathrm{i}y)$ | Replaces $z$ with $x+\mathrm{i}y$ on both sides of the equation | M1 |
| | $x-\mathrm{i}y+1+2\mathrm{i} = 4\mathrm{i}(x+\mathrm{i}y)$ or $x+\mathrm{i}y+1-2\mathrm{i} = -4\mathrm{i}(x-\mathrm{i}y)$ | Fully correct method for applying the conjugate | M1 |
| | $x-\mathrm{i}y+1+2\mathrm{i} = 4\mathrm{i}x-4y$ | | |
| | Real: $x+1=-4y$ <br> Imaginary: $-y+2=4x$ | $x+1=-4y$ and $-y+2=4x$ | A1 |
| | $4x+16y=-4$ <br> $4x+y=2$ <br> $\Rightarrow 15y=-6 \Rightarrow y=...$ | Solves two equations in $x$ and $y$ to obtain at least one of $x$ or $y$ | ddM1 |
| | So, $x=\frac{3}{5},\ y=-\frac{2}{5}$ $\left\{z=\frac{3}{5}-\frac{2}{5}\mathrm{i}\right\}$ | At least one of either $x$ or $y$ are correct | A1 |
| | | Both $x$ and $y$ are correct | A1 |
| | | | **[6]** |
| | | | **Total 12** |

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**4.** (i) The complex number $w$ is given by

$$w = \frac{p - 4\text{i}}{2 - 3\text{i}}$$

where $p$ is a real constant.

(a) Express $w$ in the form $a + b\text{i}$, where $a$ and $b$ are real constants.
Give your answer in its simplest form in terms of $p$.

**(3)**

Given that $\arg w = \frac{\pi}{4}$

(b) find the value of $p$.

**(2)**

(ii) The complex number $z$ is given by

$$z = (1 - \lambda\text{i})(4 + 3\text{i})$$

where $\lambda$ is a real constant.

Given that

$$|z| = 45$$

find the possible values of $\lambda$.
Give your answers as exact values in their simplest form.

**(3)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.** (i) | | Mark (i)(a) and (i)(b) together. | |
| | $w = \dfrac{p-4\text{i}}{2-3\text{i}} \quad \arg w = \dfrac{\pi}{4}$ | | |
| (a) **Way 1** | $w = \dfrac{(p-4\text{i})}{(2-3\text{i})} \times \dfrac{(2+3\text{i})}{(2+3\text{i})}$ | Multiplies by $\dfrac{(2+3\text{i})}{(2+3\text{i})}$ | M1 |
| . | $= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\text{i}$ | At least one of either the real or imaginary part of *w* is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone $a+\text{i}b$. | A1 |
| | | Correct *w* in its simplest form. | A1 |
| | | | **[3]** |
| (a) **Way 2** | $(a+\text{i}b)(2-3\text{i}) = (p-4\text{i})$ | | |
| | $2a+3b=p$ <br> $3a-2b=4$ | Multiplies out to obtain 2 equations in two unknowns. | M1 |
| | $= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\text{i}$ | At least one of either the real or imaginary part of *w* is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone $a+\text{i}b$. | A1 |
| | | Correct *w* in its simplest form. | A1 |
| | | | **[3]** |
| (b) | $\left\{\arg w = \dfrac{\pi}{4} \Rightarrow\right\} \quad 2p+12 = 3p-8$ o.e. seen anywhere. | Sets the numerators of the real part of their *w* equal to the imaginary part of their *w* or if arctan used, require evidence of $\tan\dfrac{\pi}{4}=1$ | M1 |
| | $\Rightarrow p = 20$ | $p = 20$ | A1 |
| | | | **[2]** |
| (ii) | $z = (1-\lambda\text{i})(4+3\text{i})$ and $\lvert z \rvert = 45$ | | |
| **Way 1** | $\sqrt{1+\lambda^2}\,\sqrt{4^2+3^2}$ | Attempts to apply $\lvert(1-\lambda\text{i})(4+3\text{i})\rvert = \sqrt{1+\lambda^2}\,\sqrt{4^2+3^2}$ | M1 |
| | $\sqrt{1+\lambda^2}\,\sqrt{4^2+3^2} = 45$ | Correct equation. | A1 |
| | $\left\{\lambda^2 = 9^2-1 \Rightarrow\right\} \; \lambda = \pm 4\sqrt{5}$ | $\lambda = \pm 4\sqrt{5}$ | A1 |
| | | | **[3]** |
| **Way 2** | $z = (4+3\lambda) + (3-4\lambda)\text{i}$ <br> $\sqrt{(4+3\lambda)^2 + (3-4\lambda)^2}$ | Attempt to multiply out, group real and imaginary parts and apply the modulus. | M1 |
| | $(4+3\lambda)^2 + (3-4\lambda)^2 = 45^2$ or <br> $\sqrt{(4+3\lambda)^2 + (3-4\lambda)^2} = 45$ | Correct equation. | A1 |
| | $\left\{16+24\lambda+9\lambda^2+9-24\lambda+16\lambda^2 = 2025\right\}$ | Condone if middle terms in expansions not explicitly stated. | |
| | $\left\{25\lambda^2 = 2000 \Rightarrow\right\} \; \lambda = \pm 4\sqrt{5}$ | $\lambda = \pm 4\sqrt{5}$ | A1 |
| | | | **[3]** <br> **8** |

| | **Question 4 Notes** | |
|---|---|---|
| (ii) | **M1** | Also allow $(1+\lambda^2)(4^2+3^2)$ for M1. |
| | **M1** | Also allow $(4+3\lambda)^2 + (3-4\lambda)^2$ for M1. |

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**4.** $$z = \frac{4}{1 + \mathrm{i}}$$

Find, in the form $a + \mathrm{i}b$ where $a, b \in \mathbb{R}$

(a) $z$

**(2)**

(b) $z^2$

**(2)**

Given that $z$ is a complex root of the quadratic equation $x^2 + px + q = 0$, where $p$ and $q$ are real integers,

(c) find the value of $p$ and the value of $q$.

**(3)**

| Question Number | Scheme | Marks |
|---|---|---|
| 4. | (a) $z = \frac{4(1-\text{i})}{(1+\text{i})(1-\text{i})}$ | M1 |
| | $z = 2(1-\text{i})$ or $2 - 2\text{i}$ or exact equivalent. | A1 (2) |
| | (b) $z^2 = (2-2\text{i})(2-2\text{i}) = 4 - 8\text{i} + 4\text{i}^2$ | M1 |
| | $= -8\text{i}$ | A1 cao (2) |
| | (c) If $z$ is a root so is $z^*$ So $(x - 2 + 2\text{i})(x - 2 - 2\text{i})$ (or $x^2 - 2\text{Re}(z).x + \|z\|^2$) | M1 |
| | So $(x - 2 + 2\text{i})(x - 2 - 2\text{i}) = 0$ (or $x^2 - 2\text{Re}(z).x + \|z\|^2 = 0$) and so $p = q =$ | M1 |
| | Equation is $x^2 - 4x + 8 (= 0)$ or $p = -4$ and $q = 8$ | A1 (3) (7 marks) |
| ALT 1 | (c)<br>Substitutes $z = 2 - 2\text{i}$ and $z^2 = -8\text{i}$ into quadratic and equates real and imaginary parts to obtain $2p + q = 0$ and $-2p - 8 = 0$ | M1 |
| | Attempts to solve simultaneous equations to obtain $p = -4$ and $q = 8$ | M1A1 |
| ALT 2 | (c)<br>Attempts to obtain $p = -$ sum of roots | M1 |
| | Attempts product of roots to obtain $q=$ | M1 |
| | Equation is $x^2 - 4x + 8 (= 0)$ or $p = -4$ and $q = 8$ | A1 |
| ALT 3 | (c) $x - 2 = \pm 2\text{i}$ either sign acceptable | M1 |
| | $(x-2)^2 = -4 \Rightarrow x^2 - 4x + 4 = -4$ i.e square and attempt to expand to give 3-term quadratic | M1 |
| | Equation is $x^2 - 4x + 8 (= 0)$ or $p = -4$ and $q = 8$ | A1 |

**Notes**

(a) M1: Multiplies numerator and denominator by $1 - \text{i}$ or by $-1 + \text{i}$
A1: cao
(b) M1: Squares their $z$, or the given $z = \frac{4}{1+\text{i}}$, to produce at least 3 terms which can be implied by the correct answer.
A1: $-8\text{i}$ or $0 - 8\text{i}$ only
(c) M1: Uses their $z$ and $z^*$ in $(x - z)(x - z^*)$
M1: Multiplies two factors and obtains $p =$ or $q =$
A1: Both correct required – can be implied by $x^2 - 4x + 8$
ALT 1
(c) M1: Substitutes their $z$ and their $z^2$ into the quadratic and equates real and imaginary parts to obtain two equations in $p$ and $q$
M1: Attempts to solve for one unknown to obtain $p =$ or $q =$
A1: Both correct required – can be implied by $x^2 - 4x + 8 (= 0)$

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**7.** A complex number $z$ is given by

$$z = a + 2\text{i}$$

where $a$ is a non-zero real number.

(a) Find $z^2 + 2z$ in the form $x + \text{i}y$ where $x$ and $y$ are real expressions in terms of $a$.
**(4)**

Given that $z^2 + 2z$ is real,

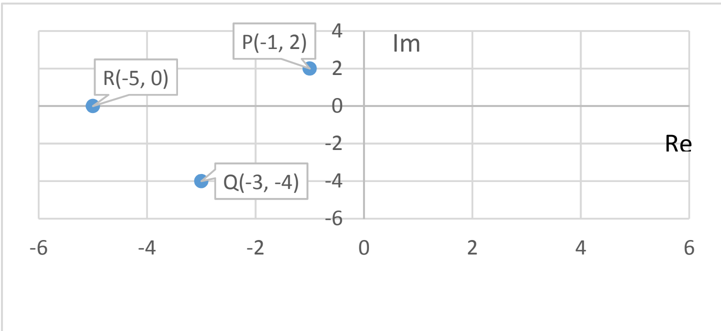
(b) find the value of $a$.
**(1)**

Using this value for $a$,

(c) find the values of the modulus and argument of $z$, giving the argument in radians, and giving your answers to 3 significant figures.
**(3)**

(d) Show the points $P$, $Q$ and $R$, representing the complex numbers $z$, $z^2$ and $z^2 + 2z$ respectively, on a single Argand diagram with origin $O$.
**(3)**

(e) Describe fully the geometrical relationship between the line segments $OP$ and $QR$.
**(2)**

| Question Number | Scheme | Marks |
|---|---|---|
| 7. | (a) $z^2 = (a+2\mathrm{i})(a+2\mathrm{i}) = (a^2-4)+4\mathrm{i}a$ | M1 |
| | So $z^2+2z = (a^2-4+2a)+\mathrm{i}(4a+4)$ or $x = (a^2+2a-4)$ and $y = 4a+4$ | M1 A1 A1 (4) |
| | (b) and so $4a+4=0 \rightarrow a=-1$ | B1 (1) |
| | ALT (b) Substitute $a=-1$ and show that $y=0$ | B1 |
| | (c) $\|z\| = \sqrt{5}$ or awrt 2.24 | B1 |
| | $\arctan,(-2) = 2.03$ | M1, A1 cao (3) |
| | (d)  | M1 A1 B1ft (3) |
| | (e) OP and QR are parallel , and QR is twice the length of OP | B1, B1 (2) |
| | Or Enlargement with Scale Factor 2 (centre O), followed by translation $\begin{pmatrix}-3\\-4\end{pmatrix}$ | **13 marks** |
| | Or Enlargement with Scale Factor 2, centre (3,4) or centre 3 + 4i | |
| | $\overrightarrow{QR} = 2\overrightarrow{OP}$ with clear indication of vectors award B1B1, without vectors award B0B1 | |

**Notes:**

(a) M1: Squares $z$ to produce at least 3 terms which can be implied by the correct answer.
M1: Adds $2z$ to their $z^2$
A1: Correct $x$ A1 Correct $y$ accept $4a\mathrm{i}+4\mathrm{i}$
(b) B1: Completely accurate cao
(c) B1: √5 or 2.24 or awrt 2.24
M1 for using tan or arctan
A1 cao 2.03
(d) M1: **Either** their $OP$ in the correct quadrant labelled $P$ or $z$ or their $-1+2\mathrm{i}$ or their (-1,2) or axes labelled **or** their $OQ$ in the correct quadrant labelled $Q$ or $z^2$ or their $-3-4\mathrm{i}$ or their (-3, -4) or axes labelled
A1: Both $OP$ and $OQ$ correct i.e. in the 2nd and 3rd quadrants respectively.
B1ft: $OR: z^2+2z(=-5)$ **on real axis** to left of the origin.
Accept points or lines. Arrows not required. Axes need not be labelled Re and Im.
Treat correct quadrant (or on axis) as important aspect for accuracy, lengths of lines if present can be accepted as correct.

Leave blank

**4.**

$$z_1 = 3\text{i} \text{ and } z_2 = \frac{6}{1 + \text{i}\sqrt{3}}$$

(a) Express $z_2$ in the form $a + \text{i}b$, where $a$ and $b$ are real numbers.

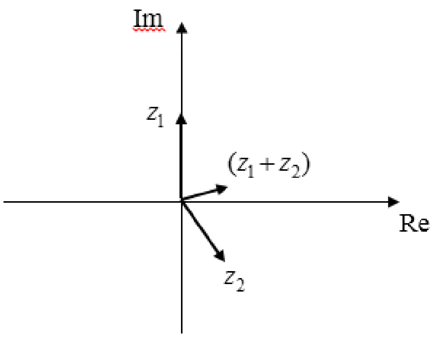
**(2)**

(b) Find the modulus and the argument of $z_2$, giving the argument in radians in terms of $\pi$.

**(4)**

(c) Show the three points representing $z_1$, $z_2$ and $(z_1 + z_2)$ respectively, on a single Argand diagram.

**(2)**

| Question Number | Scheme | Marks |
|---|---|---|
| 4. (a) | $z_2 = \dfrac{6(1-\mathrm{i}\sqrt{3})}{(1+\mathrm{i}\sqrt{3})(1-\mathrm{i}\sqrt{3})} = \dfrac{6(1-\mathrm{i}\sqrt{3})}{4}$ | M1 |
| | $z_2 = \dfrac{6(1-\mathrm{i}\sqrt{3})}{4}\left(=\dfrac{3}{2}-\mathrm{i}\dfrac{3}{2}\sqrt{3}\right)$ | A1 (2) |
| (b) | $\lvert z_2\rvert = \sqrt{\left(\dfrac{3}{2}\right)^2+\left(\dfrac{3\sqrt{3}}{2}\right)^2} = \sqrt{\dfrac{9}{4}+\dfrac{27}{4}}$ | M1 |
| | The modulus of $z_2$ is 3 | A1 |
| | $\tan\theta = (\pm)\sqrt{3}$ and attempts to find $\theta$ | M1 |
| | and the argument is $-\dfrac{\pi}{3}$ | A1 (4) |
| (c) |  | M1 A1 (2) |
| | | **(8 marks)** |



**Notes**

(a) M1: Multiplies numerator and denominator by $1-\mathrm{i}\sqrt{3}$
A1: any correct equivalent with real denominator.
(b) M1: Uses correct method for modulus for their $z_2$ in part (a)
A1: for 3 only
M1: Uses tan or inverse tan
A1: $-\dfrac{\pi}{3}$ accept $\dfrac{5\pi}{3}$
NB Answers only then award 4/4 but arg must be in terms of $\pi$
(c) M1:
**Either** $z_1$ on imaginary axis and labelled with $z_1$ or 3i or (0,3) or axis labelled 3;
**or** their $z_2$ in the correct quadrant labelled $z_2$ or $\dfrac{3}{2}-\mathrm{i}\dfrac{3}{2}\sqrt{3}$ or $\left(\dfrac{3}{2},-\dfrac{3}{2}\sqrt{3}\right)$ or axes labelled
or their $a+b\mathrm{i}$ or their $(a,b)$ or axes labelled.
Axes need not be labelled Re and Im.
A1: All 3 correct ie $z_1$ on positive imaginary axis, $z_2$ in 4th quadrant and $z_1+z_2$ in the first quadrant.
Accept points or lines. Arrows not required.