MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Polynomials - FP1 (Pearson Edexcel)

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1.
$$f(z) = 2z^3 - 4z^2 + 15z - 13$$

Given that $f(z) \equiv (z - 1)(2z^2 + az + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Hence use algebra to find the three roots of the equation $f(z) = 0$

(4)

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(z) = 2z^3 - 4z^2 + 15z - 13 \equiv (z-1)(2z^2 + az + b)$		
(a)	$a = -2,\ b = 13$	At least one of either $a = -2$ or $b = 13$ or seen as their coefficients.	B1
		Both $a = -2$ and $b = 13$ or seen as their coefficients.	B1
			[2]
(b)	$\{z =\}$ 1 is a root	1 is a root, seen anywhere.	B1
	$\left\{2z^2 - 2z + 13 = 0 \Rightarrow z^2 - z + \frac{13}{2} = 0\right\}$		
	Either • $z = \dfrac{2 \pm \sqrt{4 - 4(2)(13)}}{2(2)}$	Correct method for solving a 3-term quadratic equation. Do not allow M1 here for an attempt at factorising.	M1
	or • $\left(z - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{13}{2} = 0$ and $z = \ldots$		
	or • $(2z-1)^2 - 1 + +13 = 0$ and $z = \ldots$		
	So, $\{z =\}\ \frac{1}{2} + \frac{5}{2}\mathrm{i},\ \frac{1}{2} - \frac{5}{2}\mathrm{i}$	At least one of either $\frac{1}{2} + \frac{5}{2}\mathrm{i}$ or $\frac{1}{2} - \frac{5}{2}\mathrm{i}$ or any equivalent form.	A1
		For conjugate of first complex root	A1ft
			[4]
			Total 6

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6. Given that 4 and 2i – 3 are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation,

(1)

(b) find the value of a and the value of b.

(5)

Question Number	Scheme		Marks
6.	$x^3+ax^2+bx-52=0$, $a, b\in\mathbb{R}$, 4 and $2\text{i}-3$ are roots		
(a)	$-2\text{i}-3$	$-2\text{i}-3$ seen anywhere in solution for Q6.	B1
			[1]
(b) **Way 1**	$(x-(2\text{i}-3))(x-\text{"}(-2\text{i}-3)\text{"})$; $=x^2+6x+13$ or $x=-3\pm 2\text{i}\Rightarrow(x+3)^2=-4$; $=x^2+6x+13(=0)$ $(x-4)(x-(2\text{i}-3))$; $=x^2-(1+2\text{i})x+4(2\text{i}-3)$ $(x-4)(x-\text{"}(-2\text{i}-3)\text{"})$; $=x^2-(1-2\text{i})x+4(-2\text{i}-3)$	Must follow from their part (a). Any incorrect signs for their part (a) in initial statement award M0; accept any equivalent expanded expression for A1.	M1; A1
	$(x-4)(x^2+6x+13)$ $\{=x^3+ax^2+bx-52\}$	$(x-3^{\text{rd}}\text{ root})(\text{their quadratic})$. Could be found by comparing coefficients from long division.	M1
	$a=2,\ b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 2**	Sum $=(2\text{i}-3)+\text{"}(-2\text{i}-3)\text{"}=-6$ Product $=(2\text{i}-3)\times\text{"}(-2\text{i}-3)\text{"}=13$	Attempts to apply either $x^2-(\text{sum roots})x+(\text{product roots})=0$ or $x^2-2\text{Re}(\alpha)x+\lvert\alpha^2\rvert=0$	M1
	So quadratic is $x^2+6x+13$	$x^2+6x+13$	A1
	$(x-4)(x^2+6x+13)$ $\{=x^3+ax^2+bx-52\}$	$(x-3^{\text{rd}}\text{ root})(\text{their quadratic})$	M1
	$a=2,\ b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 3**	$(2\text{i}-3)^3+a(2\text{i}-3)^2+b(2\text{i}-3)-52=0$ $5a-3b=43$ (real parts) and $6a-b=23$ (imaginary parts) or uses f(4) = 0 and f(a complex root) = 0 to form equations in *a* and *b*.	Substitutes $2\text{i}-3$ into the displayed equation and equates both real and imaginary parts.	M1
		$5a-3b=43$ and $6a-b=23$ **or** $16a+4b=-12$ and $(2i-3)^3+a(2i-3)^2+b(2i-3)-52=0$ / $(-2i-3)^3+a(-2i-3)^2+b(-2i-3)-52=0$	A1
	So $a=2, b=-11$ or $x^3+2x^2-11x-52$	Solves these equations simultaneously to find at least one of either $a=...$ or $b=...$	M1
		At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 4**	$b=$ sum of product pairs $=4(2\text{i}-3)+4\text{"}(-2\text{i}-3)\text{"}+(2\text{i}-3)\text{"}(-2\text{i}-3)\text{"}$	Attempts sum of product pairs.	M1
		All pairs correct o.e.	A1
	$a=-(\text{sum of 3 roots})=-(4+2\text{i}\ -3\text{"}-2\text{i}-3\text{"})$	Adds up all 3 roots	M1
	$a=2,\ b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]

(b) **Way 5**	Uses f(4) = 0	M1
	$16a + 4b = -12$	A1
	$a = -(\text{sum of 3 roots}) = -(4 + 2\text{i}\ -3\text{"} - 2\text{i} - 3\text{"})$ Adds up all 3 roots	M1
	$a = 2,\ b = -11$ or $x^3 + 2x^2 - 11x - 52$ At least one of $a = 2$ or $b = -11$	A1
	Both $a = 2$ and $b = -11$	A1
		[5] **6**

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1. $$f(x) = 9x^3 - 33x^2 - 55x - 25$$

Given that $x = 5$ is a solution of the equation $f(x) = 0$, use an algebraic method to solve $f(x) = 0$ completely.

(5)

June 2015
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	$(x-5)$ is a factor of f(x) so f(x) = $(x-5)(9x^2...\quad)$	M1
	f(x) = $(x-5)(9x^2+12x+5)$	A1
	Solve $(9x^2+12x+5)=0$ to give $x=$	M1
	$(x=)-\frac{2}{3}-\frac{1}{3}\text{i},\ -\frac{2}{3}+\frac{1}{3}\text{i}$ or $-\frac{2}{3}\pm\frac{1}{3}\text{i}$ or $\frac{-2\pm\text{i}}{3}$ oe (and 5)	A1cao A1ft (5) **(5 marks)**
	Notes M1: Uses (x-5) as factor and begins division or process to obtain quadratic with $9x^2$. Award if no working but quadratic factor completely correct. A1: $9x^2+12x+5$ M1: Solves their quadratic by usual rules leading to $x=$ Award if one complex root correct with no working. Award for $(9x^2+...$ incorrectly factorised to $(3x+p)(3x+q)$, where $\lvert pq\rvert=5$ A1: One correct complex root. Accept any exact equivalent form. Accept single fraction and ± A1ft: Conjugate of their first complex root.	

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6. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}\left(5^n - 1\right) & 5^n \end{pmatrix}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n\left(4n^2 - 1\right)$$

(6)

Question Number	Scheme	Marks
6. (i)	If $n = 1$, $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^1 = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^1-1) & 5^1 \end{pmatrix}$ so **true** for ***n* = 1**	B1
	Assume result true for $n = k$	
	$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1) & 5^k \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1) & 5^k \end{pmatrix}$	M1
	$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1)-5^k & 5\times5^k \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1-5.\frac{1}{4}(5^k-1) & 5\times5^k \end{pmatrix}$	M1 A1
	$=\begin{pmatrix} 1 & 0 \\ -\frac{1}{4}5^k+\frac{1}{4}-5^k & 5^{k+1} \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1-\frac{1}{4}5^{k+1}+\frac{5}{4} & 5^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^{k+1}-1) & 5^{k+1} \end{pmatrix}$	A1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n} \in \boldsymbol{Z}^+$.	A1cso (6)
(ii)	If $n = 1$, $\sum_{r=1}^{n}(2r-1)^2 = 1$ and $\frac{1}{3}n(4n^2-1) = 1$, so **true** for ***n* = 1**.	B1
	Assume result true for $n = k$ so $\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k(4k^2-1)+(2(k+1)-1)^2$	M1
	$=\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}(2k+1)\{(2k^2-k)+(3(2k+1))\}$	M1 A1
	$=\frac{1}{3}(2k+1)\{(2k^2+5k+3)\}$ or $\frac{1}{3}(k+1)(4k^2+8k+3)$ or $\frac{1}{3}((2k+3)(2k^2+3k+1)\}$	
	$=\frac{1}{3}(k+1)(2k+1)(2k+3) \quad = \frac{1}{3}(k+1)(4(k+1)^2-1)$	dA1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n} \in \boldsymbol{Z}^+$	A1cso (6) **12 marks**

Notes

(i) B1: Checks $n = 1$ on both sides and states true for $n = 1$ seen anywhere.
M1: Assumes true for $n = k$ and indicates intention to multiply power k by power 1 either way around.
M1: Multiplies matrices. Condone one slip. A1: Correct unsimplified matrix
A1: Intermediate step required cao
A1: cso Makes correct induction statement including at least statements in bold.
Statement **true** for ***n* = 1** here could contribute to B1 mark earlier.
(ii) B1: Checks $n = 1$ on both sides and states true for $n = 1$ seen anywhere.
M1: Assumes true for $n = k$ and adds $(k+1)^{th}$ term to sum of k terms. Accept $4(k+1)^2-4(k+1)+1$ or $(2k+1)^2$ for $(k+1)^{th}$ term. M1: Factorises out a linear factor of the three possible - usually $2k+1$
A1: Correct expression with one linear and one quadratic factor.
dA1: Need to see $\frac{1}{3}(k+1)(4(k+1)^2-1)$ somewhere dependent upon previous A1.

Accept assumption plus $(k+1)^{th}$ term and $\frac{1}{3}(k+1)(4(k+1)^2-1)$ both leading to $\frac{1}{3}(4k^3+12k^2+11k+3)$ then award for expressions seen as above.
A1: cso Makes correct complete induction statement including at least statements in bold. Statement true for $n = 1$ here could contribute to B1 mark earlier.

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1. The roots of the equation

$$2z^3 - 3z^2 + 8z + 5 = 0$$

are z_1, z_2 and z_3

Given that $z_1 = 1 + 2\text{i}$, find z_2 and z_3

(5)

Question Number	Scheme		Marks
1.	$\mathrm{f}(z) = 2z^3 - 3z^2 + 8z + 5$		
	$1 - 2\mathrm{i}$ (is also a root)	seen	B1
	$(z-(1+2\mathrm{i}))(z-(1-2\mathrm{i})) = z^2 - 2z + 5$	Attempt to expand $(z-(1+2\mathrm{i}))(z-(1-2\mathrm{i}))$ or any valid method to establish the quadratic factor e.g. $z = 1 \pm 2\mathrm{i} \Rightarrow z - 1 = \pm 2\mathrm{i} \Rightarrow z^2 - 2z + 1 = -4$ $z = 1 \pm \sqrt{-4} = \frac{2 \pm \sqrt{-16}}{2} \Rightarrow b = -2, c = 5$ Sum of roots 2, product of roots 5 $\therefore z^2 - 2z + 5$	M1A1
	$\mathrm{f}(z) = (z^2 - 2z + 5)(2z + 1)$	Attempt at **linear** factor with their *cd* in $(z^2 + az + c)(2z + d) = \pm 5$ Or $(z^2 - 2z + 5)(2z + a) \Rightarrow 5a = 5$	M1
	$(z_3) = -\frac{1}{2}$		A1
			(5)
			Total 5