MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Rates and Differential Equations - C4 (Pearson Edexcel)

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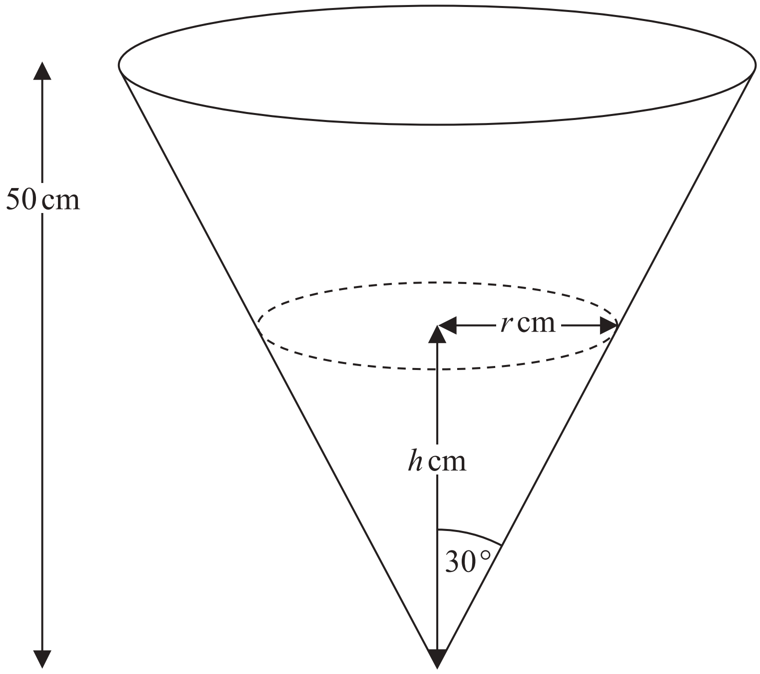

Figure 1

A water container is made in the shape of a hollow inverted right circular cone with semi-vertical angle of 30°, as shown in Figure 1. The height of the container is 50 cm.

When the depth of the water in the container is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(2)

Given that the volume of water in the container increases at a constant rate of 200 cm^3 s^{-1},

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 15$
Give your answer in its simplest form in terms of π.

(4)

Question Number	Scheme	Notes	Marks
4. (a)	$\frac{r}{h} = \tan 30 \Rightarrow r = h\tan 30 \left\{\Rightarrow r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{h}{r} = \tan 60 \Rightarrow r = \frac{h}{\tan 60} \left\{\Rightarrow r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{r}{\sin 30} = \frac{h}{\sin 60} \Rightarrow r = \frac{h\sin 30}{\sin 60} \left\{\Rightarrow r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3}h\right\}$ **or** $h^2 + r^2 = (2r)^2 \Rightarrow r^2 = \frac{1}{3}h^2$	Correct use of trigonometry to find r in terms of h **or** correct use of Pythagoras to find r^2 in terms of h^2	M1
	$\left\{V = \frac{1}{3}\pi r^2 h \Rightarrow\right\} V = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V = \frac{1}{9}\pi h^3$ *	Correct proof of $V = \frac{1}{9}\pi h^3$ or $V = \frac{1}{9}h^3\pi$ Or shows $\frac{1}{9}\pi h^3$ or $\frac{1}{9}h^3\pi$ with some reference to $V =$ in their solution	A1 *
			[2]
(b) **Way 1**	$\frac{dV}{dt} = 200$		
	$\frac{dV}{dh} = \frac{1}{3}\pi h^2$	$\frac{1}{3}\pi h^2$ o.e.	B1
	Either • $\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\} \left(\frac{1}{3}\pi h^2\right)\frac{dh}{dt} = 200$ • $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\} \frac{dh}{dt} = 200 \times \frac{1}{\frac{1}{3}\pi h^2}$	**either** $\left(\text{their } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 200$ **or** $200 \div \left(\text{their } \frac{dV}{dh}\right)$	M1
	When $h = 15,\ \frac{dh}{dt} = 200 \times \frac{1}{\frac{1}{3}\pi(15)^2} \left\{= \frac{200}{75\pi} = \frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
	$\frac{dh}{dt} = \frac{8}{3\pi}$ (cm s^{-1})	$\frac{8}{3\pi}$	A1 **cao**
			[4]
			6
(b) **Way 2**	$\frac{dV}{dt} = 200 \Rightarrow V = 200t + c \Rightarrow \frac{1}{9}\pi h^3 = 200t + c$		
	$\left(\frac{1}{3}\pi h^2\right)\frac{dh}{dt} = 200$	$\frac{1}{3}\pi h^2$ o.e.	B1
		as in Way 1	M1
	When $h = 15,\ \frac{dh}{dt} = 200 \times \frac{1}{\frac{1}{3}\pi(15)^2} \left\{= \frac{200}{75\pi} = \frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
	$\frac{dh}{dt} = \frac{8}{3\pi}$ (cm s^{-1})	$\frac{8}{3\pi}$	A1 **cao**
			[4]

		Question 4 Notes
4. (a)	**Note**	Allow M1 for writing down $r = h\tan 30$
	Note	Give M0 A0 for writing down $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$ with no evidence of using trigonometry on r and h or Pythagoras on r and h
	Note	Give M0 (unless recovered) for evidence of $\frac{1}{3}\pi r^2 h = \frac{1}{9}\pi h^3$ leading to either $r^2 = \frac{1}{3}h^2$ or $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$
(b)	**B1**	Correct simplified or un-simplified differentiation of V. E.g. $\frac{1}{3}\pi h^2$ or $\frac{3}{9}\pi h^2$
	Note	$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V
	M1	$\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 200$ or $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$
	dM1	**dependent on the previous M mark** Substitutes $h = 15$ into an expression ***which is a result*** of either $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$ or $200 \times \frac{1}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)}$
	A1	$\frac{8}{3p}$ (units are not required)
	Note	Give final A0 for using $\frac{\mathrm{d}V}{\mathrm{d}t} = -200$ to give $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{8}{3\pi}$, unless recovered to $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{8}{3\pi}$

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6. Given that $y = 2$ when $x = -\dfrac{\pi}{8}$, solve the differential equation

$$\frac{dy}{dx} = \frac{y^2}{3\cos^2 2x} \qquad -\frac{1}{2} < x < \frac{1}{2}$$

giving your answer in the form $y = f(x)$.

(6)

Question Number	Scheme	Notes	Marks
6.	$\dfrac{dy}{dx}=\dfrac{y^2}{3\cos^2 2x}$; $-\dfrac{1}{2}<x<\dfrac{1}{2}$; $y=2$ at $x=-\dfrac{\pi}{8}$		
	$\displaystyle\int \frac{1}{y^2}\,dy=\int \frac{1}{3\cos^2 2x}\,dx$	Separates variables as shown. Can be implied by a correct attempt at integration. Ignore the integral signs	B1
	$\displaystyle\int \frac{1}{y^2}\,dy=\int \frac{1}{3}\sec^2 2x\,dx$		
	$-\dfrac{1}{y}=\dfrac{1}{3}\left(\dfrac{\tan 2x}{2}\right)\{+c\}$	$\pm\dfrac{A}{y^2}\to\pm\dfrac{B}{y}$; $A, B\neq 0$	M1
		$\pm\lambda\tan 2x$	M1
		$-\dfrac{1}{y}=\dfrac{1}{3}\left(\dfrac{\tan 2x}{2}\right)$	A1
	$-\dfrac{1}{2}=\dfrac{1}{6}\tan\left(2\left(-\dfrac{\pi}{8}\right)\right)+c$	Use of $x=-\dfrac{\pi}{8}$ and $y=2$ in an integrated equation ***containing a constant of integration***, e.g. c	M1
	$-\dfrac{1}{2}=-\dfrac{1}{6}+c\Rightarrow c=-\dfrac{1}{3}$		
	$-\dfrac{1}{y}=\dfrac{1}{6}\tan 2x-\dfrac{1}{3}=\dfrac{\tan(2x)-2}{6}$		
	$y=\dfrac{-1}{\frac{1}{6}\tan 2x-\frac{1}{3}}$ or $y=\dfrac{6}{2-\tan 2x}$ or $y=\dfrac{6\cot 2x}{-1+2\cot 2x}$ $\left\{-\dfrac{1}{2}<x<\dfrac{1}{2}\right\}$		A1 o.e.
			[6]
			6

Question 6 Notes

6.	**B1**	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The number "3" may appear on either side. E.g. $\displaystyle\int \frac{1}{y^2}\,dy=\int \frac{1}{3}\sec^2 2x\,dx$ or $\displaystyle\int \frac{3}{y^2}\,dy=\int \frac{1}{\cos^2 2x}\,dx$ are fine for B1
	Note	Allow e.g. $\displaystyle\int \frac{1}{y^2}\frac{dy}{dx}\,dx=\int \frac{1}{3}\sec^2 2x\,dx$ for B1 or condone $\displaystyle\int \frac{1}{y^2}=\int \frac{1}{3}\sec^2 2x$ for B1
	Note	B1 can be implied by correct integration of both sides
	M1	$\pm\dfrac{A}{y^2}\to\pm\dfrac{B}{y}$; $A, B\neq 0$
	M1	$\dfrac{1}{\cos^2 2x}$ or $\sec^2 2x\to\pm\lambda\tan 2x$; $\lambda\neq 0$
	A1	$-\dfrac{1}{y}=\dfrac{1}{3}\left(\dfrac{\tan 2x}{2}\right)$ with or without '$+c$'. E.g. $-\dfrac{6}{y}=\tan 2x$
	M1	Evidence of using both $x=-\dfrac{\pi}{8}$ and $y=2$ in an integrated or changed equation containing c
	Note	This mark can be implied by the correct value of c
	Note	You may need to use your calculator to check that they have satisfied the final M mark
	Note	Condone using $x=\dfrac{\pi}{8}$ instead of $x=-\dfrac{\pi}{8}$
	A1	$y=\dfrac{-1}{\frac{1}{6}\tan 2x-\frac{1}{3}}$ or $y=\dfrac{6}{2-\tan 2x}$ **or any equivalent correct answer in the form** $y=f(x)$
	Note	You can ignore subsequent working, which follows from a correct answer

	Question 6 Notes Continued	
6.	**Note**	Writing $\dfrac{dy}{dx} = \dfrac{y^2}{3\cos^2 2x} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}y^2 \sec^2 2x$ leading to e.g. • $y = \dfrac{1}{9}y^3\left(\dfrac{1}{2}\tan 2x\right)$ gets 2nd M0 for $\pm\lambda\tan 2x$ • $u = \dfrac{1}{3}y^2,\ \dfrac{dv}{dx} = \sec^2 2x \Rightarrow \dfrac{du}{dx} = \dfrac{2}{3}y,\ v = \dfrac{1}{2}\tan 2x$ gets 2nd M0 for $\pm\lambda\tan 2x$ because the variables have not been separated

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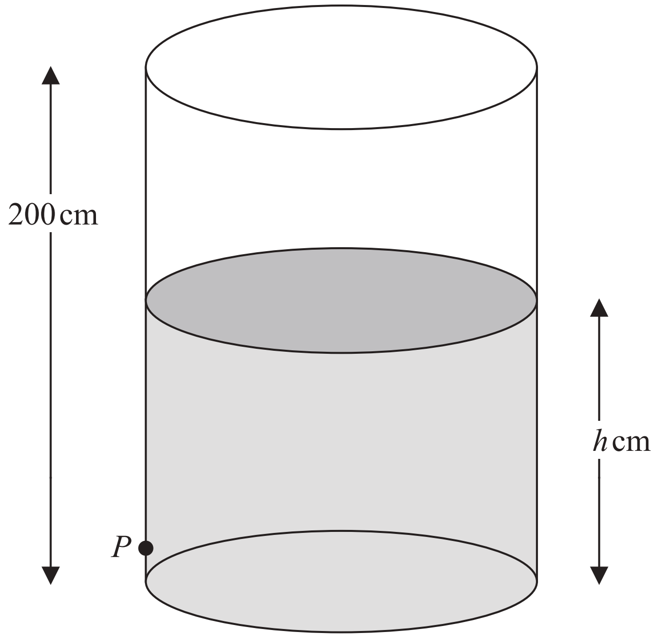

Figure 3

Figure 3 shows a vertical cylindrical tank of height 200 cm containing water.
Water is leaking from a hole P on the side of the tank.

At time t minutes after the leaking starts, the height of water in the tank is h cm.

The height h cm of the water in the tank satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = k(h-9)^{\frac{1}{2}}, \qquad 9 < h \leqslant 200$$

where k is a constant.

Given that, when $h = 130$, the height of the water is falling at a rate of 1.1 cm per minute,

(a) find the value of k.

(2)

Given that the tank was full of water when the leaking started,

(b) solve the differential equation with your value of k, to find the value of t when $h = 50$

(6)

Question Number	Scheme	Notes	Marks
7.	$\frac{\mathrm{d}h}{\mathrm{d}t} = k\sqrt{(h-9)},\ \ 9 < h \le 200;\quad h = 130,\ \frac{\mathrm{d}h}{\mathrm{d}t} = -1.1$		
(a)	$-1.1 = k\sqrt{(130-9)} \Rightarrow k = \ldots$	Substitutes $h = 130$ and either $\frac{\mathrm{d}h}{\mathrm{d}t} = -1.1$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = 1.1$ into the printed equation and rearranges to give $k = \ldots$	M1
	so, $k = -\frac{1}{10}$ or -0.1	$k = -\frac{1}{10}$ or -0.1	A1
			[2]
(b) **Way 1**	$\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int k\,\mathrm{d}t$	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Ignore the integral signs.	B1
	$\int (h-9)^{-\frac{1}{2}}\,\mathrm{d}h = \int k\,\mathrm{d}t$		
	$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\ (+c)$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = (\text{their } k)t$, with/without $+c$, or equivalent, which can be un-simplified or simplified.	A1
	$\{t = 0, h = 200 \Rightarrow\}\ 2\sqrt{(200-9)} = k(0) + c$	Some evidence of applying both $t = 0$ and $h = 200$ to changed equation containing a constant of integration, e.g. c or A	M1
	$\Rightarrow c = 2\sqrt{191} \Rightarrow 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$ $\{h = 50 \Rightarrow\}\ 2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$ $t = \ldots$	**dependent on the previous M mark** Applies $h = 50$ and their value of c to their changed equation and rearranges to find the value of $t = \ldots$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145\ldots = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ isw or awrt 148	A1 cso
			[6]
(b) **Way 2**	$\int_{200}^{50} \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int_0^T k\,\mathrm{d}t$	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits not necessary.	B1
	$\int_{200}^{50} (h-9)^{-\frac{1}{2}}\,\mathrm{d}h = \int_0^T k\,\mathrm{d}t$		
	$\left[\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{200}^{50} = [kt]_0^T$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = (\text{their } k)t$, with/without limits, or equivalent, which can be un-simplified or simplified.	A1
	$2\sqrt{41} - 2\sqrt{191} = kt$ or kT	Attempts to apply limits of $h = 200, h = 50$ and (can be implied) $t = 0$ to their changed equation	M1
	$t = \frac{2\sqrt{41} - 2\sqrt{191}}{-0.1}$	**dependent on the previous M mark** Then rearranges to find the value of $t = \ldots$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145\ldots = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ **or** awrt 148 **or** 2 hours and awrt 28 minutes	A1 cso
			[6]
			8

	Question 7 Notes	
7. (b)	**Note**	Allow first B1 for writing $\dfrac{\mathrm{d}t}{\mathrm{d}h} = \dfrac{1}{k\sqrt{(h-9)}}$ or $\dfrac{\mathrm{d}t}{\mathrm{d}h} = \dfrac{1}{(\text{their } k)\sqrt{(h-9)}}$ or equivalent
	Note	$\dfrac{\mathrm{d}t}{\mathrm{d}h} = \dfrac{1}{k\sqrt{(h-9)}}$ leading to $t = \dfrac{2}{k}\sqrt{(h-9)}\ (+c)$ with/without $+c$ is B1M1A1
	Note	After finding $k = 0.1$ in part (a), it is only possible to gain full marks in part (b) by **initially writing** $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{(h-9)}$ **or** $\displaystyle\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int k\,\mathrm{d}t$ **or** $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -0.1\sqrt{(h-9)}$ **or** $\displaystyle\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int 0.1\mathrm{d}t$ Otherwise, those candidates who find $k = 0.1$ in part (a), should lose at least the final A1 mark in part (b).

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4. The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{5}{2}x, \qquad t \geqslant 0$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams. Give your answer to the nearest minute.

(3)

Question Number	Scheme	Notes	Marks
4.	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 1**	$\int \frac{1}{x}\,\mathrm{d}x = \int -\frac{5}{2}\,\mathrm{d}t$	Separates variables as shown. $\mathrm{d}x$ and $\mathrm{d}t$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 2**	$\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,\mathrm{d}x$	**Either** $\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ **or** $t = \int -\frac{2}{5x}\,\mathrm{d}x$	B1
	$t = -\frac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = \ldots$ **or** $\pm\alpha\ln px;\ \alpha \neq 0,\ p > 0$	M1
		$t = -\frac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ c = \frac{2}{5}\ln 60 \Rightarrow t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 3**	$\int_{60}^{x} \frac{1}{x}\,\mathrm{d}x = \int_{0}^{t} -\frac{5}{2}\,\mathrm{d}t$	Ignore limits	B1
	$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]
(b)	$20 = 60\mathrm{e}^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda\mathrm{e}^{\pm\mu t} \pm\beta$ **or** $x = \pm\lambda\mathrm{e}^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm\beta$ **or** $t = \pm\lambda\ln\delta x \pm\beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449\ldots \text{(days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ($A \in$ [illegible], $t > 0$)	dM1
	$\Rightarrow t = 632.8006\ldots = 633$ (to the nearest minute)	awrt 633 **or** 10 hours and awrt 33 minutes	A1 **cso**
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 4**	$\int \frac{2}{5x}\,dx = -\int dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\frac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ \frac{2}{5}\ln 300 = c$ $\frac{2}{5}\ln(5x) = -t + \frac{2}{5}\ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 5**	$\left\{\frac{dt}{dx} = -\frac{2}{5x} \Rightarrow\right\}\ t = \int_{60}^{x} -\frac{2}{5x}\,dx$	Ignore limits	B1
	$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]

		Question 4 Notes
4. (a)	**B1**	For the correct separation of variables. E.g. $\int \frac{1}{5x}\,dx = \int -\frac{1}{2}\,dt$
	Note	B1 can be implied by seeing **either** $\ln x = -\frac{5}{2}t + c$ **or** $t = -\frac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \frac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \to x = 60e^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen.
(b)	**A1**	You can apply **cso** for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0

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7. (a) Express $\dfrac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \quad t \geqslant 0$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time. Give your answer in years to 3 significant figures.

(3)

Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\dfrac{dP}{dt} = \dfrac{1}{2}P(P-2)\cos 2t$		
	$\displaystyle\int \frac{2}{P(P-2)}\,dP = \int \cos 2t\,dt$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t\ \ (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P$, $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\quad \ln 1 - \ln 3 = 0 + c\quad \{\Rightarrow\ c = -\ln 3\ \text{or}\ \ln(\tfrac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$ $\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$		
	$\dfrac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$ gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \dfrac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right)\ \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] **13**

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
7. (b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \ \ (+c)$	As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0, P = 3 \Rightarrow\}\quad 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow\ 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	**A1 * cso**

	Question 7 Notes	
7. (a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer ***cannot*** be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1,\ B = 1$

7. (b)	**B1**	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Eg:** $\int \frac{2}{P^2-2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ **o.e.** are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0,\ \mu \neq 0.$ Also allow $\pm\lambda\ln(M(P-2)) \pm \mu\ln NP;\ M, N$ can be 1.
	Note	Condone $2\ln(P-2) + 2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2-2P)$ or $\ln(P^2-2P)$
	1st A1	Correct result of $\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$ or $2\ln(P-2) - 2\ln P = \sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t=0$ and $P=3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,\ \lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. c, A, $\ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P = \frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$ followed by $\frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} + \mathrm{e}^c$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \rightarrow \frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t + c} \rightarrow \frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} + \mathrm{e}^c$ is final M1M0A0
		<u>**4th M1 for making *P* the subject**</u> **Note there are three type of manipulations here which are considered acceptable for making *P* the subject.** **(1)** M1 for $\frac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P(3-\mathrm{e}^{\frac{1}{2}\sin 2t}) = 6$ $\Rightarrow P = \frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$ **(2)** M1 for $\frac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \frac{6}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \mathrm{e}^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow\Rightarrow P = \frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$ **(3)** M1 for $\left\{\ln(P-2) + \ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow\right\} P(P-2) = 3\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P^2 - 2P = 3\mathrm{e}^{\frac{1}{2}\sin 2t}$ $\Rightarrow (P-1)^2 - 1 = 3\mathrm{e}^{\frac{1}{2}\sin 2t}$ leading to $P = ..$
(c)	**M1**	States $P=4$ or applies $P=4$
	M1	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, where λ and k are numerical values and λ can be 1
	A1	anything that rounds to 0.473. ***(Do not apply isw here)***
	Note	***Do not apply ignore subsequent working for A1.*** (Eg: 0.473 followed by 473 years is A0.)
	Note	<u>Use of $P = 4000$</u>: Without the mention of $P=4$, $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
	Note	<u>Use of Degrees:</u> $t =$ awrt 27.1 will usually imply M1M1A0