MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Vectors - C4 (Pearson Edexcel)

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7. The point A with coordinates $(-3, 7, 2)$ lies on a line l_1
The point B also lies on the line l_1

Given that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$,

(a) find the coordinates of point B.
(2)

The point P has coordinates $(9, 1, 8)$

(b) Find the cosine of the angle PAB, giving your answer as a simplified surd.
(3)

(c) Find the exact area of triangle PAB, giving your answer in its simplest form.
(3)

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2
(2)

The point Q lies on the line l_2

Given that the line segment AP is perpendicular to the line segment BQ,

(e) find the coordinates of the point Q.
(5)

Question Number	Scheme	Notes	Marks
7.	$\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}, \overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}, \overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}; \overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$	Let $\theta=$ size of angle PAB. A, B lie on l_1 and P lies on l_2	
(a)	$\left\{\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}\Rightarrow\right\}$	Attempts to add $\overrightarrow{OA}$ to $\overrightarrow{AB}$	M1
	$\overrightarrow{OB}=\begin{pmatrix}-3\\7\\2\end{pmatrix}+\begin{pmatrix}4\\-6\\2\end{pmatrix}=\begin{pmatrix}1\\1\\4\end{pmatrix}\Rightarrow B(1, 1, 4)$	$(1, 1, 4)$ or $\begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\mathbf{i}+\mathbf{j}+4\mathbf{k}$	A1
	Note: M1 can be implied by at least 2 correct components for B		**[2]**
(b)	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	$\left\{\cos\theta=\dfrac{\overrightarrow{AP}\bullet\overrightarrow{AB}}{\lvert\overrightarrow{AP}\rvert\lvert\overrightarrow{AB}\rvert}\right\}=\dfrac{\begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$	Applies dot product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ or a multiple of these vectors	dM1
	$\left\{\cos\theta=\dfrac{96}{\sqrt{216}.\sqrt{56}}\Rightarrow\cos\theta\right\}=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]
(c)	$\left\{\cos\theta=\dfrac{4}{\sqrt{21}}\right\}\Rightarrow\sin\theta=\dfrac{\sqrt{21-16}}{\sqrt{21}}=\dfrac{\sqrt{5}}{\sqrt{21}}=\dfrac{\sqrt{105}}{21}$	A correct method for converting an exact value for $\cos q$ to an exact value for $\sin q$	M1
	Area $PAB=\dfrac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)\ \left\{=12\sqrt{21}\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)\right\}=12\sqrt{5}$	see notes	M1
		$12\sqrt{5}$	A1 **cao**
			[3]
(d)	$\{l_2:\}\ \mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\1\end{pmatrix}$	$\mathbf{p}+\lambda\mathbf{d}$ or $\mathbf{p}+\mu\mathbf{d}$, $\mathbf{p}\neq 0$, $\mathbf{d}\neq 0$ with either $\mathbf{p}=9\mathbf{i}+\mathbf{j}+8\mathbf{k}$ or $\mathbf{d}=4\mathbf{i}-6\mathbf{j}+2\mathbf{k}$ or $\mathbf{d}=$ multiple of $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$	M1
		Correct vector equation	A1
			[2]
(e)	$\overrightarrow{BQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\right\}\ \left\{\overrightarrow{QB}=\begin{pmatrix}-8-4\mu\\6\mu\\-4-2\mu\end{pmatrix}\right\}$	Applies their $\overrightarrow{OQ}-$ their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}-$ their $\overrightarrow{OQ}$	M1
	$\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$	Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for μ	dM1
	$\Rightarrow 96+48\mu+36\mu+24+12\mu=0\Rightarrow 96\mu+120=0\Rightarrow\mu=-\dfrac{5}{4}$	$\mu=-\dfrac{120}{96}$ or $\mu=-\dfrac{5}{4}$	A1 o.e.
	$\overrightarrow{OQ}=\begin{pmatrix}9+4(-1.25)\\1-6(-1.25)\\8+2(-1.25)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4, 8.5, 5.5)$	Substitutes their value of μ into $\overrightarrow{OQ}$	ddM1
		$(4, 8.5, 5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$	A1 o.e.
			[5]
			15

Question Number	Scheme	Notes	Marks
7.	$\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}, \overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}, \overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}; \overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$	Let $\theta=$ size of angle PAB. A, B lie on l_1 and P lies on l_2	
(e) **Alt 1**	$\overrightarrow{BQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\right\}$ $\left\{\overrightarrow{QB}=\begin{pmatrix}-8-2\mu\\3\mu\\-4-\mu\end{pmatrix}\right\}$	Applies their $\overrightarrow{OQ}-$ their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}-$ their $\overrightarrow{OQ}$	M1
	$\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$	Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for μ	dM1
	$\Rightarrow 96+24\mu+18\mu+24+6\mu=0\Rightarrow 48\mu+120=0\Rightarrow\mu=-\frac{5}{2}$	$\mu=-\frac{5}{2}$	A1 o.e.
	$\overrightarrow{OQ}=\begin{pmatrix}9+2(-2.5)\\1-3(-2.5)\\8+1(-2.5)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4, 8.5, 5.5)$	Substitutes their value of μ into $\overrightarrow{OQ}$	ddM1
		$(4, 8.5, 5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$	A1 o.e.
			[5]
(b) **Alt 1**	**<u>Vector Cross Product:</u>** Use this scheme if a vector cross product method is being applied		
	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	$\mathbf{d}_1\times\mathbf{d}_2=\underline{\begin{pmatrix}12\\-6\\6\end{pmatrix}\times\begin{pmatrix}4\\-6\\2\end{pmatrix}}=\left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\12&-6&6\\4&-6&2\end{vmatrix}=24\mathbf{i}+0\mathbf{j}-48\mathbf{k}\right\}$		
	$\sin\theta=\dfrac{\sqrt{(24)^2+(0)^2+(-48)^2}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$	Applies vector cross product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ or a multiple of these vectors	dM1
	$\left\{\sin\theta=\dfrac{\sqrt{2880}}{\sqrt{216}.\sqrt{56}}=\sqrt{\dfrac{5}{21}}\right\}\{\Rightarrow\cos\theta\}=\sqrt{\dfrac{16}{21}}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]
(b) **Alt 2**	**<u>Cosine Rule</u>**		
	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	Note: $\lvert\overrightarrow{PA}\rvert=\sqrt{216}$, $\lvert\overrightarrow{AB}\rvert=\sqrt{56}$ and $\lvert\overrightarrow{PB}\rvert=\sqrt{80}$		
	$(\sqrt{80})^2=(\sqrt{216})^2+(\sqrt{56})^2-2(\sqrt{216})(\sqrt{56})\cos\theta$	Applies the cosine rule the correct way round	dM1
	$\cos\theta=\dfrac{216+56-80}{2\sqrt{216}\sqrt{56}}=\dfrac{192}{2\sqrt{216}\sqrt{56}}$		
	$\{\Rightarrow\cos\theta\}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]

	Question 7 Notes	
7. (b)	**Note**	If no "subtraction" seen, you can award 1st M1 for 2 out of 3 correct components of the difference
	Note	For dM1 the dot product formula can be applied as $\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}\cos\theta = \begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}$
	Note	***Evaluation*** of the dot product for $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is not required for the dM1 mark
	A1	For either $\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ or $\cos\theta=\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$
	Note	Using $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\frac{24+18+6}{\sqrt{216}.\sqrt{14}}=\frac{48}{12\sqrt{21}}=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Using $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\frac{4+3+1}{\sqrt{6}.\sqrt{14}}=\frac{8}{2\sqrt{21}}=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Give M1M1A0 for finding $\theta=$ awrt 29.2 without reference to $\cos\theta=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Condone taking the dot product between vectors the wrong way round for the M1 dM1 marks
	Note	**<u>Vectors the wrong way round</u>**
		• E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\frac{4}{\sqrt{21}}$ or $-\frac{4}{21}\sqrt{21}$ ***with no other working*** is final A0 • E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\frac{4}{\sqrt{21}}$ or $-\frac{4}{21}\sqrt{21}$ **followed by** $\cos\theta=\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ or just simply writing $\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ is final A1
	Note	In part (b), give M0dM0 for finding and using $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$
(c)	**Note**	Give 1st M0 for $\sin\theta=\sin\left(\cos^{-1}\left(\frac{4\sqrt{21}}{21}\right)\right)$ or $\sin\theta=1-\left(\frac{4}{21}\sqrt{21}\right)^2$ ***unless recovered***
	M1	Give 2nd M1 for either • $\frac{1}{2}$(their length AP)(their length AB)(their attempt at $\sin\theta$) • $\frac{1}{2}$(their length AP)(their length AB)sin(their 29.2° from part (b)) • $\frac{1}{2}$(their length AP)(their length AB)$\sin\theta$; where $\cos\theta=...$ in part (b)
	Note	$\frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)$sin(awrt 29.2° or awrt 150.8°) {= awrt 26.8} without reference to finding $\sin\theta$ as an exact value if M0 M1 A0
	Note	Anything that rounds to 26.8 without reference to finding $\sin\theta$ as an exact value is M0 M1 A0
	Note	Anything that rounds to 26.8 without reference to $12\sqrt{5}$ is A0
	Note	If they use $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$ in part (b), then this can be followed through in part (c) for the 2nd M mark as e.g. $\frac{1}{2}\left(\sqrt{110}\right)\left(\sqrt{56}\right)\sin\theta$
	Note	Finding $12\sqrt{5}$ in part (c) is M1 dM1 A1, even if there is little or no evidence of finding an exact value for $\sin\theta$. So $\frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\sin(29.2°)=12\sqrt{5}$ is M1 dM1 A1

		Question 7 Notes Continued
7. (d)	**Note**	Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is not required for the M mark
	A1	Writing $\mathbf{r} = \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}2\\-3\\1\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d} =$ a multiple of $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
	Note	Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is required for the A mark
	Note	Other valid $\mathbf{p} = \begin{pmatrix}9\\1\\8\end{pmatrix}$ are e.g. $\mathbf{p} = \begin{pmatrix}13\\-5\\10\end{pmatrix}$ or $\mathbf{p} = \begin{pmatrix}5\\7\\6\end{pmatrix}$. So $\mathbf{r} = \begin{pmatrix}13\\-5\\10\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ is M1 A1
	Note	Give A0 for writing $l_2 : \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or ans $= \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ unless recovered
	Note	Using scalar parameter λ or other scalar parameters (e.g. μ or s or t) is fine for M1 and/or A1
(e)	**ddM1**	Substitutes their value of μ into $\overrightarrow{OQ}$, where $\overrightarrow{OQ}$ = their equation for l_2
	Note	If they use $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{AB} = (5\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$ in part (b), then this can be followed through in part (e) for the 2nd M mark and the 3rd M mark
	Note	You imply the final M mark in part (e) for at least 2 correctly followed through components for Q from their μ

Question Number	Scheme	Notes	Marks
7. (c) **Alt 1**	**Vector Cross Product:** Use this scheme if a vector cross product method is being applied		
	$\overrightarrow{AP} \times \overrightarrow{AB} = \begin{pmatrix}12\\-6\\6\end{pmatrix} \times \begin{pmatrix}4\\-6\\2\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\12 & -6 & 6\\4 & -6 & 2\end{vmatrix} = 24\mathbf{i} + 0\mathbf{j} - 48\mathbf{k}\right\}$		
	Area $PAB = \frac{1}{2}\sqrt{(24)^2 + (-48)^2}$	Uses a vector product and $\sqrt{("24")^2 + ("0")^2 + ("-48")^2}$	M1
		Uses a vector product and $\frac{1}{2}\sqrt{("24")^2 + ("0")^2 + ("-48")^2}$	M1
	$= 12\sqrt{5}$	$12\sqrt{5}$	A1 **cao**
			[3]
7. (c) **Alt 2**	**Note:** $\cos APB = \frac{5}{\sqrt{30}}$ or $\frac{1}{6}\sqrt{30}$ **Note:** $\lvert\overrightarrow{PA}\rvert = \sqrt{216}$ and $\lvert\overrightarrow{PB}\rvert = \sqrt{80}$		
	$\sin\theta = \frac{\sqrt{30-25}}{\sqrt{30}} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{6}}{6}$	A correct method for converting an exact value for $\cos q$ to an exact value for $\sin q$	M1
	Area $PAB = \frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{80}\right)\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\left\{= 12\sqrt{30}\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\right\} = 12\sqrt{5}$	$\frac{1}{2}$(their PA)(their PB)$\sin\theta$	M1
		$12\sqrt{5}$	A1 **cao**
			[3]

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6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

where λ and μ are scalar parameters.

The lines l_1 and l_2 intersect at the point X.

(a) Find the coordinates of the point X.

(3)

(b) Find the size of the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point A lies on l_1 and has position vector $\begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}$

(c) Find the distance AX, giving your answer as a surd in its simplest form.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YA}$ is perpendicular to the line l_1

(d) find the distance YA, giving your answer to one decimal place.

(2)

The point B lies on l_1 where $|\overrightarrow{AX}| = 2|\overrightarrow{AB}|$.

(e) Find the two possible position vectors of B.

(3)

Question Number	Scheme	Notes	Marks
6.	$l_1 : \mathbf{r} = \begin{pmatrix}4\\28\\4\end{pmatrix} + \lambda\begin{pmatrix}-1\\-5\\1\end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix}5\\3\\1\end{pmatrix} + \mu\begin{pmatrix}3\\0\\-4\end{pmatrix}$; $\overrightarrow{OA} = \begin{pmatrix}2\\18\\6\end{pmatrix}$ lies on l_1	Let θ_{Acute} be the acute angle between l_1 and l_2	
(a)	$\{l_1 = l_2 \Rightarrow\}$ $28 - 5\lambda = 3$ $\{\Rightarrow \lambda = 5\}$ or $4 - \lambda = 5 + 3\mu$ and $4 + \lambda = 1 - 4\mu$ $\{\Rightarrow \mu = -2\}$	$28 - 5\lambda = 3$ or $4 - \lambda = 5 + 3\mu$ and $4 + \lambda = 1 - 4\mu$ or $\lambda = 5$ or $\mu = -2$ (Can be implied).	B1
	$\{\overrightarrow{OX} =\}$ $\begin{pmatrix}4\\28\\4\end{pmatrix} + 5\begin{pmatrix}-1\\-5\\1\end{pmatrix}$ or $\begin{pmatrix}5\\3\\1\end{pmatrix} - 2\begin{pmatrix}3\\0\\-4\end{pmatrix}$	Puts $l_1 = l_2$ and solves to find λ and/or μ **and** substitutes their value for λ into l_1 or their value for μ into l_2	M1
	So, $X(-1, 3, 9)$	$(-1, 3, 9)$ or $\begin{pmatrix}-1\\3\\9\end{pmatrix}$ or $-\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$ or condone $\begin{matrix}-1\\3\\9\end{matrix}$	A1 **cao**
			[3]
(b) **Way 1**	$\mathbf{d}_1 = \begin{pmatrix}-1\\-5\\1\end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix}3\\0\\-4\end{pmatrix}$ $\Rightarrow \begin{pmatrix}-1\\-5\\1\end{pmatrix} \bullet \begin{pmatrix}3\\0\\-4\end{pmatrix}$	Realisation that the dot product is required between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	M1
	$\cos\theta = \dfrac{\pm\begin{pmatrix}-1\\-5\\1\end{pmatrix}\bullet\begin{pmatrix}3\\0\\-4\end{pmatrix}}{\sqrt{(-1)^2+(-5)^2+(1)^2}\,.\sqrt{(3)^2+(0)^2+(-4)^2}}$ $\left\{= \dfrac{-7}{\sqrt{27}.\sqrt{25}}\right\}$	**dependent on the 1st M mark.** Applies dot product formula between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	dM1
	$\{\theta = 105.6303588... \Rightarrow\}$ $\theta_{\text{Acute}} = 74.36964117... = 74.37$ (2 dp)	awrt 74.37 seen in (b) only	A1
			[3]
(c)	$\overrightarrow{AX} = "\overrightarrow{OX}" - \overrightarrow{OA} = \begin{pmatrix}-1\\3\\9\end{pmatrix} - \begin{pmatrix}2\\18\\6\end{pmatrix} = \begin{pmatrix}-3\\-15\\3\end{pmatrix}$ or $A_{\lambda=2}, X_{\lambda=5} \Rightarrow AX = 3\lvert\mathbf{d}_1\rvert$, $\left\{\lvert\mathbf{d}_1\rvert = \sqrt{27}\right\}$		
	$AX = \sqrt{(-3)^2 + (-15)^2 + (3)^2}$ or $3\sqrt{27}$ $\left\{= \sqrt{243}\right\} = 9\sqrt{3}$	Full method for finding AX or XA	M1
		$9\sqrt{3}$ seen in (c) only	A1 **cao**
	Note: You cannot recover work for part (c) in either part (d) or part (e).		**[2]**
(d) **Way 1**	$\dfrac{YA}{"9\sqrt{3}"} = \tan("74.36964...")$	$\dfrac{YA}{\text{their } \lvert\overrightarrow{AX}\rvert} = \tan\theta$ or $YA = \left(\text{their } \lvert\overrightarrow{AX}\rvert\right)\tan\theta$, where θ is their acute or obtuse angle between l_1 and l_2	M1
	$YA = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
(e) **Way 1**	$\{A_{\lambda=2}, X_{\lambda=5} \Rightarrow$ So $AX = 2AB \Rightarrow$ So at B, $\lambda = 3.5$ or $\lambda = 0.5)\}$		
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 3.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Substitutes **either** $\lambda = \dfrac{(\text{their } \lambda_X \text{ found in } (a)) + 2}{2}$ **or** $\lambda_b = 3 - \dfrac{(\text{their } \lambda_X \text{ found in } (a))}{2}$ into l_1	M1;
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 0.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct. (Also allow coordinates).	A1
		Both position vectors are correct. (Also allow coordinates).	A1
			[3]
			13

Question Number	Scheme	Notes	Marks
6. (e) **Way 2**	$\left\{AX = 2AB \Rightarrow AB = \frac{1}{2}AX. \text{ So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \overrightarrow{AB} \Rightarrow \overrightarrow{OB} = \overrightarrow{OA} \pm \frac{1}{2}\overrightarrow{AX}\right\}$		
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} + 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Applies either $\overrightarrow{OA} + 0.5\overrightarrow{AX}$ **or** $\overrightarrow{OA} - 0.5\overrightarrow{AX}$ where (their $\overrightarrow{AX}$) $= \pm\left[(\text{their } \overrightarrow{OX}) - \overrightarrow{OA}\right]$	M1;
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]
6. (e) **Way 3**	$\overrightarrow{AB} = \begin{pmatrix}4-\lambda\\28-5\lambda\\4+\lambda\end{pmatrix} - \begin{pmatrix}2\\18\\6\end{pmatrix} = \begin{pmatrix}2-\lambda\\10-5\lambda\\-2+\lambda\end{pmatrix} = \begin{pmatrix}1(2-\lambda)\\5(2-\lambda)\\-1(2-\lambda)\end{pmatrix}$; $\overrightarrow{AX} = \begin{pmatrix}-3\\-15\\3\end{pmatrix}$ $AX^2 = 243 \Rightarrow$ $AB^2 = 27(2-\lambda)^2$ $AX = 2AB \Rightarrow AX^2 = 4AB^2 \Rightarrow 243 = 4(27)(2-\lambda)^2 \Rightarrow (2-\lambda)^2 = \frac{9}{4}$ **or** $27\lambda^2 - 108\lambda + \frac{189}{4} = 0$ **or** $108\lambda^2 - 432\lambda + 189 = 0$ **or** $4\lambda^2 - 16\lambda + 7 = 0 \Rightarrow \lambda = 3.5$ or $\lambda = 0.5$		
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 3.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Full method of solving for λ the equation $AX^2 = 4AB^2$ using (their $\overrightarrow{AX}$) and $\overrightarrow{AB}$ **and** substitutes at least one of their values for λ into l_1	M1;
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 0.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
	Note: $AX = 2AB \Rightarrow \overrightarrow{AX} = \pm 2\overrightarrow{AB}$. Hence, $\lambda = 3.5$ or $\lambda = 0.5$ can be found from solving either $x: -3 = \pm 2(2-\lambda)$ or $y: -15 = \pm 2(10-5\lambda)$ or $z: -3 = \pm 2(-2+\lambda)$		**[3]**
6. (e) **Way 4**	$\overrightarrow{OB} = \begin{pmatrix}-1\\3\\9\end{pmatrix} + 0.5\begin{pmatrix}3\\15\\-3\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Applies **either** (their $\overrightarrow{OX}$) $+ 0.5\overrightarrow{XA}$ **or** (their $\overrightarrow{OX}$) $+ 1.5\overrightarrow{XA}$ where (their $\overrightarrow{XA}$) $= \overrightarrow{OA} -$ (their $\overrightarrow{OX}$)	M1;
	$\overrightarrow{OB} = \begin{pmatrix}-1\\3\\9\end{pmatrix} + 1.5\begin{pmatrix}3\\15\\-3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]
6. (e) **Way 5**	$\overrightarrow{OB} = 0.5\left(\begin{pmatrix}-1\\3\\9\end{pmatrix} + \begin{pmatrix}2\\18\\6\end{pmatrix}\right); = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Applies $\frac{1}{2}\left[(\text{their } \overrightarrow{OX}) + \overrightarrow{OA}\right]$	M1;
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]

Question Number	Scheme	Notes	Marks
6. (e) **Way 6**	$\left\{ \lvert\overrightarrow{AX}\rvert = 9\sqrt{3}\,,\ \lvert d_1\rvert = 3\sqrt{3} \Rightarrow K = \frac{9\sqrt{3}}{3\sqrt{3}} = 3 \Rightarrow \overrightarrow{AX} = 3\mathbf{d}_1\text{; So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \tfrac{1}{2}\overrightarrow{AX} = \overrightarrow{OA} \pm \tfrac{1}{2}(3\mathbf{d}_1) \right\}$		
	$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} + 0.5\left(3\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}\right); = \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$	Applies either $\overrightarrow{OA} + 0.5(K\mathbf{d}_1)$ **or** $\overrightarrow{OA} - 0.5(K\mathbf{d}_1)$, where $K = \frac{\text{their } \lvert\overrightarrow{AX}\rvert}{3\sqrt{3}}$	M1;
	$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - 0.5\left(3\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}\right); = \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]

Question 6 Notes

6. (a)	**Note**	M1 can be implied by at least two correct follow through coordinates from their $/$ or from their m
(b)	**Note**	***Evaluating*** the dot product (i.e. $(-1)(3) + (-5)(0) + (1)(-4)$) is not required for the M1, dM1 marks.
	Note	**For M1 dM1:** Allow one slip in writing down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$
	Note	Allow M1 dM1 for $\left(\sqrt{(-1)^2 + (-5)^2 + (1)^2}\,.\sqrt{(3)^2 + (0)^2 + (-4)^2}\right)\cos q = \pm \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$
	Note	$q = 1.297995...^c$, (without evidence of awrt 74.37) is A0

6. (b) **Way 2**	<u>**Alternative Method: Vector Cross Product**</u>		
	Only apply this scheme if it is clear that a vector cross product method is being applied.		
	$\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -5 & 1 \\ 3 & 0 & -4 \end{vmatrix} = 20\mathbf{i} - \mathbf{j} + 15\mathbf{k} \right\}$	Realisation that the vector cross product is required between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	M1
	$\sin q = \frac{\sqrt{(20)^2 + (-1)^2 + (15)^2}}{\sqrt{(-1)^2 + (-5)^2 + (1)^2}\,.\sqrt{(3)^2 + (0)^2 + (-4)^2}}$	Applies the vector product formula between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	dM1
	$\sin q = \frac{\sqrt{626}}{\sqrt{27}\,.\sqrt{25}}$ Þ $q = 74.36964117... = 74.37$ (2 dp)	awrt 74.37 seen in (b) only	A1
			[3]

6. (c)	**M1**	Finds the difference between their $\overrightarrow{OX}$ and $\overrightarrow{OA}$ and applies Pythagoras to the result to find AX or XA **OR** applies $\left\lvert\left(\text{their } /_X \text{ found in } (a)\right) - 2\right\rvert.\sqrt{(-1)^2 + (-5)^2 + (1)^2}$
	Note	For M1: Allow one slip in writing down their $\overrightarrow{OX}$ and $\overrightarrow{OA}$
	Note	Allow M1A1 for $\begin{pmatrix} 3 \\ 15 \\ 3 \end{pmatrix}$ leading to $AX = \sqrt{(3)^2 + (15)^2 + (3)^2} = \sqrt{243} = 9\sqrt{3}$
(e)	**Note**	Imply M1 for no working leading to any two components of one of the $\overrightarrow{OB}$ which are correct.

Question Number	Scheme	Notes	Marks
6. (d) **Way 2**	$\dfrac{"9\sqrt{3}"}{YA} = \tan(90 - "74.36964...")$	$\dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{YA} = \tan(90-\theta)$ or $AY = \dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{\tan(90-\theta)}$, where θ is the acute or obtuse angle between l_1 and l_2	M1
	$YA = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
6. (d) **Way 3**	$\dfrac{YA}{\sin("74.36964...")} = \dfrac{"9\sqrt{3}"}{\sin(90 - "74.36964...")}$	$\dfrac{YA}{\sin\theta} = \dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{\sin(90-\theta)}$ o.e., where θ is the acute or obtuse angle between l_1 and l_2	M1
	$YA = \dfrac{9\sqrt{3}\sin(74.36964...)}{\sin(15.63036...)} = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
6. (d) **Way 4**	$\mathbf{d}_1 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}, \quad \overrightarrow{OY} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix}$		
	$\overrightarrow{YA} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix} = \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix}$ $\overrightarrow{YA} \bullet \mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} = 0$ $\Rightarrow 3+3\mu-75+5+4\mu=0 \Rightarrow \mu = \dfrac{67}{7}$ $YA^2 = \left(-3-3\left(\dfrac{67}{7}\right)\right)^2 + (15)^2 + \left(5+4\left(\dfrac{67}{7}\right)\right)^2$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{YA} \bullet \mathbf{d}_1 = 0$ or $\overrightarrow{AY} \bullet \mathbf{d}_1 = 0$ or $\overrightarrow{YA} \bullet (K\mathbf{d}_1) = 0$ or $\overrightarrow{AY} \bullet (K\mathbf{d}_1) = 0$ to find μ and applies Pythagoras to find a numerical expression for AY^2 or for the distance AY	M1
	So, $YA = \sqrt{\left(-\dfrac{222}{7}\right)^2 + (15)^2 + \left(\dfrac{303}{7}\right)^2}$		
	$= 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
	Note: $\overrightarrow{OY} = \dfrac{236}{7}\mathbf{i} + 3\mathbf{j} - \dfrac{261}{7}\mathbf{k}$, $\overrightarrow{AY} = -\dfrac{222}{7}\mathbf{i} + 15\mathbf{j} + \dfrac{303}{7}\mathbf{k}$		**[2]**

Leave blank

8. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$.

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.
Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(2)

The acute angle between AP and l_2 is θ.

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2
Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

Question Number	Scheme	Notes	Marks
8.	$l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$. $\overrightarrow{OA}$ occurs when $\mu = 1$. $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$		
(a)	$A(3, 5, 0)$	$(3, 5, 0)$	B1
			[1]
(b)	$\{l_2 :\}\ \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	$\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} + t\mathbf{d}$, $\mathbf{a} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$	A1
	$\mathbf{d}_2$ is the direction vector of l_2	Do not allow $l_2 :$ **or** $l_2 \rightarrow$ **or** $l_1 =$ for the A1 mark.	**[2]**
(c)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$		
	$AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$	Full method for finding AP	M1
		$2\sqrt{2}$	A1
			[2]
(d)	So $\overrightarrow{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	Realisation that the dot product is required between $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AP} \bullet \mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert.\lvert\mathbf{d}_2\rvert} = \dfrac{\pm\left(\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}\right)}{\sqrt{(-2)^2 + (0)^2 + (2)^2}.\sqrt{(-5)^2 + (4)^2 + (3)^2}}$	**dependent on the previous M mark.** Applies dot product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\{\cos\theta\} = \dfrac{\pm(10+0+6)}{\sqrt{8}.\sqrt{50}} = \dfrac{4}{5}$	$\{\cos\theta\} = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$	A1 cso
			[3]
(e)	$\{\text{Area } APE =\}\ \dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$	$\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$ or $\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin(\text{their } \theta)$	M1
	$= 2.4$	2.4 or $\dfrac{12}{5}$ or $\dfrac{24}{10}$ or awrt 2.40	A1
			[2]
(f)	$\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE =$ their $2\sqrt{2}$ from part (c)		
	$\{PE^2 =\}\ (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$	This mark can be implied.	M1
	$\left\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \dfrac{4}{25} \Rightarrow\right\}\ \lambda = \pm\dfrac{2}{5}$	Either $\lambda = \dfrac{2}{5}$ or $\lambda = -\dfrac{2}{5}$	A1
	$l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \pm \dfrac{2}{5}\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	**dependent on the previous M mark** Substitutes at least one of their values of λ into l_2.	dM1
	$\{\overrightarrow{OE}\} = \begin{pmatrix} 3 \\ \frac{17}{5} \\ \frac{4}{5} \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 3.4 \\ 0.8 \end{pmatrix}$, $\{\overrightarrow{OE}\} = \begin{pmatrix} -1 \\ \frac{33}{5} \\ \frac{16}{5} \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 6.6 \\ 3.2 \end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5]
			15

		Question 8 Notes
8. (a)	**B1**	Allow $A(3, 5, 0)$ or $3\mathbf{i}+5\mathbf{j}$ or $3\mathbf{i}+5\mathbf{j}+0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$
(b)	**A1**	Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 = i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$.
	Note	Allow the use of parameters μ or t instead of λ.
(c)	**M1**	Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find AP
	Note	Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2+(0)^2+(2)^2} = \sqrt{8} = 2\sqrt{2}$.
(d)	**Note**	For both the M1 and dM1 marks $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) must be the vector used in part (c) or the difference $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ from part (a).
	Note	Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1
	Note	***Evaluating*** the dot product (i.e. $(-2)(-5)+(0)(4)+(2)(3)$) is not required for M1 and dM1 marks.
	Note	In part (d) allow one slip in writing $\overrightarrow{AP}$ and $\mathbf{d}_2$
	Note	$\cos\theta = \dfrac{-10+0-6}{\sqrt{8}.\sqrt{50}} = -\dfrac{4}{5}$ followed by $\cos\theta = \dfrac{4}{5}$ is fine for A1 cso
	Note	Give M1dM1A1 for $\{\cos\theta =\} = \dfrac{\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-10\\8\\6\end{pmatrix}}{\sqrt{8}.10\sqrt{2}} = \dfrac{20+12}{40} = \dfrac{4}{5}$
	Note	Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by $36.869...^\circ$
	<u>Alternative Method: Vector Cross Product</u>	
	Only apply this scheme if it is clear that a candidate is applying a vector cross product method.	
	$\underline{\overrightarrow{AP}\times\mathbf{d}_2 = \begin{pmatrix}-2\\0\\2\end{pmatrix}\times\begin{pmatrix}-5\\4\\3\end{pmatrix}} = \left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-2&0&2\\-5&4&3\end{vmatrix} = -8\mathbf{i}-4\mathbf{j}-8\mathbf{k}\right\}$	Realisation that the vector cross product is required between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ — M1
	$\sin\theta = \dfrac{\sqrt{(-8)^2+(-4)^2+(-8)^2}}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$	Applies the vector product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ — dM1
	$\sin\theta = \dfrac{12}{\sqrt{8}.\sqrt{50}} = \dfrac{3}{5} \Rightarrow \underline{\cos\theta = \dfrac{4}{5}}$	$\cos\theta = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$ — A1
(e)	**Note**	Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2\sin(36.869...^\circ)$ or $\frac{1}{2}(2\sqrt{2})^2\sin(180^\circ - 36.869...^\circ)$; = awrt 2.40
	Note	Candidates must use their θ from part (d) or apply a correct method of finding their $\sin\theta = \dfrac{3}{5}$ from their $\cos\theta = \dfrac{4}{5}$

	Question 8 Notes Continued	
8. (f)	**Note**	Allow the first M1A1 for deducing $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ from no incorrect working
	SC	Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working
	Note	Give 1st M1 for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$
	Note	Give 1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent
	Note	Give 1st M1 for $\lambda = \dfrac{\text{their } AP = \text{"}2\sqrt{2}\text{"}}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1st A1 for $\lambda = \dfrac{2\sqrt{2}}{5\sqrt{2}}$
	Note	So $\left\{\hat{\mathbf{d}}_1 = \frac{1}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow\right\}$ "vector" $= \frac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ is M1A1
	Note	The 2nd dM1 in part (f) can be implied for at least 2 (out of 6) correct *x*, *y*, *z* ordinates from their values of λ.
	Note	Giving their "coordinates" as a column vector or position vector is fine for the final A1A1.
	CAREFUL	Putting l_2 equal to A gives $\begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}3\\5\\0\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = \frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$ — Give M0 dM0 for finding and using $\lambda = \frac{2}{5}$ from this incorrect method.
	CAREFUL	Putting $\lambda\mathbf{d}_2 = \overrightarrow{AP}$ gives $\lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}2\\0\\-2\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = -\frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$ — Give M0 dM0 for finding and using $\lambda = -\frac{2}{5}$ from this incorrect method.
	General	You can follow through the part (c) answer of their $AP = 2\sqrt{2}$ for (d) M1dM1, (e) M1, (f) M1dM1
	General	You can follow through their $\mathbf{d}_2$ in part (b) for (d) M1dM1, (f) M1dM1.

Leave blank

4. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A.

(a) Find the coordinates of A.
(2)

(b) Find the value of the constant p.
(3)

(c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.
(3)

The point B lies on l_2 where $\mu = 1$

(d) Find the shortest distance from the point B to the line l_1, giving your answer to 3 significant figures.
(3)

Question Number	Scheme		Marks
4.	$l_1: \mathbf{r} = \begin{pmatrix}5\\-3\\p\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\-3\end{pmatrix}$, $l_2: \mathbf{r} = \begin{pmatrix}8\\5\\-2\end{pmatrix} + \mu\begin{pmatrix}3\\4\\-5\end{pmatrix}$. Let $\theta =$ acute angle between l_1 and l_2. **Note: You can mark parts (a) and (b) together.**		
(a)	$\{l_1 = l_2 \Rightarrow \mathbf{i}:\}\ 5 = 8 + 3\mu \Rightarrow \mu = -1$	Finds μ and substitutes their μ into l_2	M1
	So, $\{\overrightarrow{OA}\} = \begin{pmatrix}8\\5\\-2\end{pmatrix} - 1\begin{pmatrix}3\\4\\-5\end{pmatrix} = \begin{pmatrix}5\\1\\3\end{pmatrix}$	$5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix}5\\1\\3\end{pmatrix}$ or $(5, 1, 3)$	A1
			[2]
(b)	$\{\mathbf{j}:\ -3 + \lambda = 5 + 4\mu \Rightarrow\}\ -3 + \lambda = 5 + 4(-1) \Rightarrow \lambda = 4$	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$	M1
	$\mathbf{k}:\ p - 3\lambda = -2 - 5\mu \Rightarrow$ $p - 3(4) = -2 - 5(-1) \Rightarrow \underline{p = 15}$ or $\mathbf{k}:\ p - 3\lambda = 3 \Rightarrow$ $p - 3(4) = 3 \Rightarrow \underline{p = 15}$	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p - 3\lambda =$ the **k** value of A found in part (a)", substitutes their λ and solves to give $p = ...$	M1
		$p = 15$	A1
			[3]
(c)	$\mathbf{d}_1 = \begin{pmatrix}0\\1\\-3\end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix}3\\4\\-5\end{pmatrix} \Rightarrow \begin{pmatrix}0\\1\\-3\end{pmatrix} \bullet \begin{pmatrix}3\\4\\-5\end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm K\left(\dfrac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2}\,.\sqrt{(3)^2 + (4)^2 + (-5)^2}}\right)$	An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	dM1 **(A1 on ePEN)**
	$\cos\theta = \dfrac{19}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1
			[3]
(d)	$\overrightarrow{OB} = \begin{pmatrix}11\\9\\-7\end{pmatrix}$; $\overrightarrow{AB} = \begin{pmatrix}11\\9\\-7\end{pmatrix} - \begin{pmatrix}5\\1\\3\end{pmatrix} = \begin{pmatrix}6\\8\\-10\end{pmatrix}$ or $\overrightarrow{AB} = 2\begin{pmatrix}3\\4\\-5\end{pmatrix} = \begin{pmatrix}6\\8\\-10\end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{6^2 + 8^2 + (-10)^2}\ \{= 10\sqrt{2}\}$	**See notes**	M1
	$\dfrac{d}{10\sqrt{2}} = \sin\theta$	Writes down a correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$, oe.	dM1
	$\{d = 10\sqrt{2}\sin 31.82... \Rightarrow\}\ d = 7.456540753... = 7.46$ (3sf)	anything that rounds to 7.46	A1
			[3] **11**

4. (b)	**Alternative method for part (b)** $\begin{cases} 3\times \mathbf{j}: & -9+3\lambda = 15+12\mu \\ \mathbf{k}: & p-3\lambda = -2+5\mu \end{cases}$ $p-9=13+7\mu$	Eliminates λ to write down an equation in p and μ	M1
	$p-9=13+7(-1) \Rightarrow \underline{p=15}$	Substitutes their μ and solves to give $p = ...$	M1
		$p=15$	A1
4. (d)	**Alternative Methods for part (d)** Let X be the foot of the perpendicular from B onto l_1		
	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $\overrightarrow{OX} = \begin{pmatrix} 5 \\ -3 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix}$		
	$\overrightarrow{BX} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix}$		
	Method 1		
	$\overrightarrow{BX} \bullet \mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = -12+\lambda-66+9\lambda = 0$ leading to $10\lambda - 78 = 0 \Rightarrow \lambda = \frac{39}{5}$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{BX} \bullet \mathbf{d}_1 = 0$ and solves the resulting equation to find a value for λ.	M1
	$\overrightarrow{BX} = \begin{pmatrix} -6 \\ -12+\frac{39}{5} \\ 22-3\left(\frac{39}{5}\right) \end{pmatrix} = \begin{pmatrix} -6 \\ -\frac{21}{5} \\ -\frac{7}{5} \end{pmatrix}$	Substitutes their value of λ into their $\overrightarrow{BX}$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{(-6)^2 + \left(-\frac{21}{5}\right)^2 + \left(-\frac{7}{5}\right)^2} = 7.456540753...$	awrt 7.46	A1
	Method 2		
	Let $\beta = \left\|\overrightarrow{BX}\right\|^2 = 36 + 144 - 24\lambda + \lambda^2 + 484 - 132\lambda + 9\lambda^2$ $= 10\lambda^2 - 156\lambda + 664$ So $\frac{\mathrm{d}\beta}{\mathrm{d}\lambda} = 20\lambda - 156 = 0 \Rightarrow \lambda = \frac{39}{5}$	Finds $\beta = \left\|\overrightarrow{BX}\right\|^2$ in terms of λ, finds $\frac{\mathrm{d}\beta}{\mathrm{d}\lambda}$ and sets this result equal to 0 and finds a value for λ.	M1
	$\left\|\overrightarrow{BX}\right\|^2 = 10\left(\frac{39}{5}\right)^2 - 156\left(\frac{39}{5}\right) + 664 = \frac{278}{5}$	Substitutes their value of λ into their $\left\|\overrightarrow{BX}\right\|^2$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{\frac{278}{5}} = 7.456540753...$	awrt 7.46	A1

		Question 4 Notes
4. (a)	**M1**	Finds μ and substitutes their μ into l_2
	A1	Point of intersection of $5\mathbf{i}+\mathbf{j}+3\mathbf{k}$. Allow $\begin{pmatrix}5\\1\\3\end{pmatrix}$ or $(5, 1, 3)$.
	Note	You cannot recover the answer for part (a) in part (c) or part (d).
(b)	**M1**	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$
	M1	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p-3\lambda=$ the **k** value of A" found in part (b).
	A1	$p = 15$
(c)	**NOTE**	**Part (c) appears as M1A1A1 on ePEN, but now is marked as M1M1A1.**
	M1	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	Note	Allow one slip in candidates copying down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	A1	anything that rounds to 31.82. This can also be achieved by $180 - 148.1796... =$ awrt 31.82
	Note	$\theta = 0.5553...^c$ is A0.
	Note	**M1A1 for** $\cos\theta = \left(\dfrac{0-16-60}{\sqrt{(0)^2+(4)^2+(-12)^2}\,.\sqrt{(-3)^2+(-4)^2+(5)^2}}\right) = \dfrac{-76}{\sqrt{160}.\sqrt{50}}$

Alternative Method: Vector Cross Product

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$\mathbf{d}_1\times\mathbf{d}_2 = \begin{pmatrix}0\\1\\-3\end{pmatrix}\times\begin{pmatrix}3\\4\\-5\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&1&-3\\3&4&-5\end{vmatrix} = 7\mathbf{i}-9\mathbf{j}-3\mathbf{k}\right\}$	Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
$\sin\theta = \dfrac{\sqrt{(7)^2+(-9)^2+(3)^2}}{\sqrt{(0)^2+(1)^2+(-3)^2}\,.\sqrt{(3)^2+(4)^2+(-5)^2}}$	An attempt to apply the vector cross product formula	dM1 **(A1 on ePEN)**
$\sin\theta = \dfrac{\sqrt{139}}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1

(d)	**M1**	Full method for finding B **and** for finding the magnitude of $\overrightarrow{AB}$ or the magnitude of $\overrightarrow{BA}$.
	dM1	**dependent on the first method mark being awarded.** Writes down correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$ **or** $\dfrac{d}{\text{their } AB} = \cos(90-\theta)$, o.e., where " their AB" is a value. and $\theta =$ "their θ" or stated as θ
	A1	anything that rounds to 7.46

Leave blank

6. With respect to a fixed origin, the point A with position vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ lies on the line l_1 with equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \text{where } \lambda \text{ is a scalar parameter,}$$

and the point B with position vector $4\mathbf{i} + p\mathbf{j} + 3\mathbf{k}$, where p is a constant, lies on the line l_2 with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}, \quad \text{where } \mu \text{ is a scalar parameter.}$$

(a) Find the value of the constant p.

(1)

(b) Show that l_1 and l_2 intersect and find the position vector of their point of intersection, C.

(4)

(c) Find the size of the angle ACB, giving your answer in degrees to 3 significant figures.

(3)

(d) Find the area of the triangle ABC, giving your answer to 3 significant figures.

(2)

Question Number	Scheme		Marks
6.	$l_1 : \mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-1\end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix}7\\0\\7\end{pmatrix} + \mu\begin{pmatrix}3\\-5\\4\end{pmatrix}$ $\overrightarrow{OA} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}4\\p\\3\end{pmatrix}$	A lies on l_1 and B lies on l_2	
(a)	$\{B \text{ lies on } l_2 \Rightarrow \mu = -1 \Rightarrow\}$ $p = 5$	$p = 5$	B1
			[1]
(b)	$\{l_1 = l_2 \Rightarrow\} \begin{cases}\mathbf{i}: & 1 = 7 + 3\mu\\ \mathbf{j}: & 2 + 2\lambda = -5\mu\\ \mathbf{k}: & 3 - \lambda = 7 + 4\mu\end{cases}$		
	e.g. $\mathbf{i}$: $7 + 3\mu = 1$	Writes down an equation involving only one parameter.	M1
	So, $\mu = -2$	$\mu = -2$	A1
	Point of intersection is $\overrightarrow{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}$	$\mathbf{i} + 10\mathbf{j} - \mathbf{k}$	B1
	Finds $\lambda = 4$ and either • checks $\lambda = 4$ and $\mu = -2$ is true for the third component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$		B1
			[4]
(b)	**<u>Alternative Method:</u>** Solving $\mathbf{j}$ and $\mathbf{k}$ simultaneously gives		
	$8 = 14 + 3\mu$ or $23 + 3\lambda = 35$	Writes down an equation involving only one parameter.	M1
	So, $\mu = -2$ or $\lambda = 4$	Either $\mu = -2$ or $\lambda = 4$	A1
	Point of intersection is $\overrightarrow{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}$	$\mathbf{i} + 10\mathbf{j} - \mathbf{k}$	B1
	Finds $\lambda = 4$ and either • checks $\mu = -2$ is true for the $\mathbf{i}$ component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$		B1
			[4]
(c)	$\overrightarrow{AC} = \begin{pmatrix}1\\10\\-1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}0\\8\\-4\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}1\\10\\-1\end{pmatrix} - \begin{pmatrix}4\\5\\3\end{pmatrix} = \begin{pmatrix}-3\\5\\-4\end{pmatrix}$	An attempt to find both the vectors $(\overrightarrow{AC}$ or $\overrightarrow{CA})$ **and** $(\overrightarrow{BC}$ or $\overrightarrow{CB})$.	M1
	$\cos ACB = \dfrac{\overrightarrow{AC} \bullet \overrightarrow{BC}}{\lvert\overrightarrow{AC}\rvert . \lvert\overrightarrow{BC}\rvert} = \dfrac{\pm\left(\begin{pmatrix}0\\8\\-4\end{pmatrix} \bullet \begin{pmatrix}-3\\5\\-4\end{pmatrix}\right)}{\sqrt{(0)^2 + (8)^2 + (-4)^2} . \sqrt{(-3)^2 + (5)^2 + (-4)^2}}$	Applies dot product formula between their $(\overrightarrow{AC}$ or $\overrightarrow{CA})$ **and** their $(\overrightarrow{BC}$ or $\overrightarrow{CB})$.	M1
	$\left\{\cos ACB = \dfrac{0 + 40 + 16}{\sqrt{80}.\sqrt{50}} = \dfrac{56}{\sqrt{4000}} \Rightarrow\right\}$ $ACB = 27.69446... = 27.7$ (3 sf)	Anything that rounds to 27.7	A1
			[3]
(d)	Area $ACB = \dfrac{1}{2}(\sqrt{80})(\sqrt{50})\sin 27.69446...^\circ = 14.696888...$	See notes	M1
		Anything that rounds to 14.7	A1
			[2] **10**

	Question 6: Alternative Methods for Part (c)		
6. (c)	**<u>Alternative Method 1: Using the direction vectors of Line 1 and Line 2</u>**		
	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$		
	$\cos\theta = \dfrac{\mathbf{d}_1 \bullet \mathbf{d}_1}{\lvert\mathbf{d}_1\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}}{\sqrt{(0)^2+(2)^2+(-1)^2}\,.\sqrt{(3)^2+(-5)^2+(4)^2}}$	Applies dot product formula between their $\mathbf{d}_1$ and $\mathbf{d}_2$	M2
	$\left\{\cos\theta = \dfrac{0-10-4}{\sqrt{5}.\sqrt{50}} = \dfrac{-7\sqrt{10}}{25} \Rightarrow\right\} \theta = 152.3054385...$		
	Angle $ACB = 180 - 152.3054385... = 27.69446145... = 27.7 \text{ (3 sf)}$	Anything that rounds to 27.7	A1
			[3]
	<u>Alternative Method 2: The Cosine Rule</u>		
	$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	Also $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$		
	Note $\lvert\overrightarrow{AC}\rvert = \sqrt{80}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{50}$ and $\lvert\overrightarrow{AB}\rvert = \sqrt{18}$		
	$\left(\sqrt{18}\right)^2 = \left(\sqrt{80}\right)^2 + \left(\sqrt{50}\right)^2 - 2\left(\sqrt{80}\right)\left(\sqrt{50}\right)\cos\theta$	Applies the cosine rule the correct way round.	M1 oe
	$\left\{\cos\theta = \dfrac{7\sqrt{10}}{25}\right\} \Rightarrow \theta = 27.69446145... = 27.7 \text{ (3 sf)}$	Anything that rounds to 27.7	A1
			[3]
	<u>Alternative Method 3: Vector Cross Product</u>		
	Only apply this scheme if it is clear that a candidate is applying a vector cross product method.		
	$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\overrightarrow{AC} \times \overrightarrow{BC} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \times \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -4 \\ -3 & 5 & -4 \end{vmatrix} = 24\mathbf{i} + 12\mathbf{j} + 24\mathbf{k} \right\}$	Full method for applying the vector cross product formula between their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** their $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\sin ACB = \dfrac{\sqrt{(24)^2+(12)^2+(12)^2}}{\sqrt{(0)^2+(8)^2+(-4)^2}\,.\sqrt{(-3)^2+(5)^2+(-4)^2}}$		
	$\left\{\sin ACB = \dfrac{\sqrt{864}}{\sqrt{80}.\sqrt{50}} = \dfrac{3\sqrt{15}}{25} \Rightarrow\right\} \theta = 27.69446145... = 27.7 \text{ (3 sf)}$	Anything that rounds to 27.7	A1
			[3]

		Question 6 Notes
6. (a)	**B1**	$p = 5$ (Ignore working.)
(b)		**Method 1**
	M1	Writes down an equation involving only one parameter. This equation will usually be $7 + 3\mu = 1$ which is found from equating the **i** components of l_1 and l_2.
	A1	Finds $\mu = -2$
	B1	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$. Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$.
	B1	Finds $\lambda = 4$ and either • checks $\lambda = 4$ and $\mu = -2$ is true for the third component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$
(b)		**Alternative Method**
	M1	Writes down an equation involving only one parameter. Solving the **j** and **k** components simultaneously will usually give either $8 = 14 + 3\mu$ or $23 + 3\lambda = 35$
	A1	Finds either $\mu = -2$ or $\lambda = 4$
	B1	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$. Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$.
	B1	Finds $\lambda = 4$ and either • checks $\mu = -2$ is true for the **i** component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$
(c)	**M1**	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$ by subtracting.
	M1	Applies dot product **formula** between their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** their $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.
	A1	anything that rounds to 27.7
	Note	An answer of 0.48336... in radians without the correct answer in degrees is A0.
	Note	Some candidates will apply the dot product formula between vectors which are the wrong way round and achieve $152.3054385...^\circ$. If they give the acute equivalent of awrt 27.7 then award A1.
(d)	**M1**	$\frac{1}{2}(\text{their length } AC)(\text{their length } BC)\sin(\text{their } 27.7^\circ \text{ from part (c)})$
	A1	anything that rounds to 14.7. Also allow $6\sqrt{6}$.
	Note	Area $ACB = \frac{1}{2}(\sqrt{80})(\sqrt{50})\sin(152.3054385...^\circ) =$ awrt 14.7 is M1A1.