MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Integration - C4 (Pearson Edexcel)

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3. (i) Given that

$$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find

$$\int \frac{13-4x}{(2x+1)^2(x+3)}\,\mathrm{d}x, \quad x > -\frac{1}{2}$$

(3)

(ii) Find

$$\int (\mathrm{e}^x + 1)^3\,\mathrm{d}x$$

(3)

(iii) Using the substitution $u^3 = x$, or otherwise, find

$$\int \frac{1}{4x + 5x^{\frac{1}{3}}}\,\mathrm{d}x, \quad x > 0$$

(4)

Question Number	Scheme	Notes	Marks
3. (i)	$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$		
(a)	$B=6,\ C=1$	At least one of $B=6$ or $C=1$	B1
		Both $B=6$ and $C=1$	B1
	$13-4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$ $x=-3 \Rightarrow 25 = 25C \Rightarrow C=1$ $x=-\frac{1}{2} \Rightarrow 13--2 = \frac{5}{2}B \Rightarrow 15 = 2.5B \Rightarrow B=6$	Writes down a correct identity and attempts to find the value of either one of A or B or C	M1
	Either $x^2: 0 = 2A + 4C$, constant: $13 = 3A + 3B + C$, $x: -4 = 7A + B + 4C$ or $x=0 \Rightarrow 13 = 3A + 3B + C$ leading to $A=-2$	Using a correct identity to find $A=-2$	A1
			[4]
(b)	$\int \frac{13-4x}{(2x+1)^2(x+3)}\,dx = \int \frac{-2}{(2x+1)} + \frac{6}{(2x+1)^2} + \frac{1}{(x+3)}\,dx$		
	$= \frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ o.e. $\{= -\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3)\ \{+c\}\}$	See notes	M1
		At least two terms correctly integrated	A1ft
		Correct answer, o.e. Simplified or un-simplified. The correct answer must be stated on one line Ignore the absence of '$+c$'	A1
			[3]
(ii)	$\{(e^x+1)^3 =\}\ e^{3x} + 3e^{2x} + 3e^x + 1$	$e^{3x} + 3e^{2x} + 3e^x + 1$, simplified or un-simplified	B1
	$\left\{\int (e^x+1)^3\,dx\right\} = \frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x\ \{+c\}$	At least 3 examples (see notes) of correct ft integration	M1
		$\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x$, simplified or un-simplified with or without $+c$	A1
			[3]
(iii)	$\int \frac{1}{4x+5x^{\frac{1}{3}}}\,dx,\ \ x>0;\ \ u^3 = x$		
	$3u^2\frac{du}{dx} = 1$	$3u^2\frac{du}{dx} = 1$ or $\frac{dx}{du} = 3u^2$ or $\frac{du}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ or $3u^2du = dx$ o.e.	B1
	$= \int \frac{1}{4u^3+5u}.3u^2\,du\ \ \left\{= \int \frac{3u}{4u^2+5}\,du\right\}$	Expression of the form $\int \frac{\pm ku^2}{4u^3 \pm 5u}\{du\}$, $k \neq 0$ Does not have to include integral sign or du Can be implied by later working	M1
	$= \frac{3}{8}\ln(4u^2+5)\ \{+c\}$	**dependent on the previous M mark** $\pm\lambda\ln(4u^2+5)$; λ is a constant; $\lambda \neq 0$	dM1
	$= \frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\ \{+c\}$	Correct answer in x with or without $+c$	A1
			[4]
			14

	Question 3 Notes		
3. (iii) **Alt 1**	**Alternative method 1 for part (iii)**		
	$\left\{\int \frac{1}{4x+5x^{\frac{1}{3}}}\,dx\right\} = \int \frac{x^{-\frac{1}{3}}}{4x^{\frac{2}{3}}+5}\,dx$	Attempts to multiply numerator and denominator by $x^{-\frac{1}{3}}$	M1
		Expression of the form $\int \frac{\pm kx^{-\frac{1}{3}}}{4x^{\frac{2}{3}} \pm 5}\,dx,\ k \neq 0$ Does not have to include integral sign or du Can be implied by later working	M1
	$= \frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\{+c\}$	$\pm\lambda\ln(4x^{\frac{2}{3}}+5)$; λ is a constant; $\lambda \neq 0$	dM1
		Correct answer in x with or without $+c$	A1
			[4]

3. (i) (a)	**M1**	Writes down ***a correct identity*** (although this can be implied) and attempts ***to find the value of at least one*** of either A or B or C. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients.
	Note	The correct partial fraction from no working scores B1B1M1A1
(i) (b)	**M1**	At least 2 of either $\pm\frac{P}{(2x+1)} \to \pm D\ln(2x+1)$ or $\pm D\ln(x+\frac{1}{2})$ or $\pm\frac{Q}{(2x+1)^2} \to \pm E(2x+1)^{-1}$ or $\pm\frac{R}{(x+3)} \to \pm F\ln(x+3)$ for their constants P, Q, R.
	A1ft	At least two terms from any of $\pm\frac{P}{(2x+1)}$ or $\pm\frac{Q}{(2x+1)^2}$ or $\pm\frac{R}{(x+3)}$ correctly integrated.
	Note	Can be un-simplified for the A1ft mark.
	A1	Correct answer of $\frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ simplified or un-simplified. with or without '+ c'.
	Note	Allow final A1 for equivalent answers, e.g. $\ln\left(\frac{x+3}{2x+1}\right) - \frac{3}{2x+1}\{+c\}$ or $\ln\left(\frac{2x+6}{2x+1}\right) - \frac{3}{2x+1}\{+c\}$
	Note	**Beware that** $\int \frac{-2}{(2x+1)}\,dx = \int \frac{-1}{(x+\frac{1}{2})}\,dx = -\ln(x+\frac{1}{2})\{+c\}$ is correct integration
	Note	E.g. Allow M1 A1ft A1 for a correct un-simplified $\ln(x+3) - \ln(x+\frac{1}{2}) - \frac{3}{2}(x+\frac{1}{2})^{-1}\ \{+c\}$
	Note	Condone 1st A1ft for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $-\ln 2x+1 - 3(2x+1)^{-1} + \ln x+3\ \{+c\}$ unless recovered
(ii)	**Note**	Give B1 for an un-simplified $e^{3x} + 2e^{2x} + e^{2x} + 2e^{x} + e^{x} + 1$
	M1	At least 3 of either $ae^{3x} \to \frac{a}{3}e^{3x}$ **or** $be^{2x} \to \frac{b}{2}e^{2x}$ **or** $de^{x} \to de^{x}$ **or** $\mu \to \mu x$; $\alpha, \beta, \delta, \mu \neq 0$
	Note	Give A1 for an un-simplified $\frac{1}{3}e^{3x} + e^{2x} + \frac{1}{2}e^{2x} + 2e^{x} + e^{x} + x$, with or without $+c$
(iii)	**Note**	1st M1 can be implied by $\int \frac{\pm ku}{4u^2 \pm 5}\{du\},\ k \neq 0$. Does not have to include integral sign or du
	Note	Condone 1st M1 for expressions of the form $\int\left(\frac{\pm 1}{4u^3 \pm 5u}.\frac{\pm k}{u^{-2}}\right)\{du\},\ k \neq 0$
	Note	Give 2nd M0 for $\frac{3u}{8u}\ln(4u^2+5)\ \{+c\}$ (u's not cancelled) unless recovered in later working
	Note	E.g. Give 2nd M0 for integration leading to $\frac{3}{4}u\ln(4u^2+5)$ as this is not in the form $\pm\lambda\ln(4u^2+5)$

	Note	Condone 2nd M1 for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $\frac{3}{8}\ln 4x^{\frac{2}{3}} + 5\ \{+c\}$ unless recovered

Question Number	Scheme	Notes	Marks
3. (ii) **Alt 1**	$\int (e^x+1)^3\,dx;\quad u = e^x + 1 \Rightarrow \frac{du}{dx} = e^x$		
	$\left\{= \int \frac{u^3}{(u-1)}\,du = \right\} \int \left(u^2 + u + 1 + \frac{1}{u-1}\right) du$	$\int \left(u^2 + u + 1 + \frac{1}{u-1}\right)\{du\}$ where $u = e^x + 1$	B1
	$= \frac{1}{3}u^3 + \frac{1}{2}u^2 + u + \ln(u-1)\ \{+c\}$	At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u-1} \to \lambda\ln(u-1)$; $\alpha, \beta, \delta, \lambda \neq 0$	M1
	$= \frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + \ln(e^x+1-1)\ \{+c\}$		
	$= \frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + x\ \{+c\}$	$\frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + x$ or $\frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + e^x + x$ simplified or un-simplified with or without $+c$ **Note:** $\ln(e^x+1-1)$ needs to be simplified to x for this mark	A1
			[3]
3. (ii) **Alt 2**	$\int (e^x+1)^3\,dx;\quad u = e^x \Rightarrow \frac{du}{dx} = e^x$		
	$\left\{= \int \frac{(u+1)^3}{u}\,du = \right\} \int \left(u^2 + 3u + 3 + \frac{1}{u}\right) du$	$\int \left(u^2 + 3u + 3 + \frac{1}{u}\right)\{du\}$ where $u = e^x$	B1
	$= \frac{1}{3}u^3 + \frac{3}{2}u^2 + 3u + \ln u\ \{+c\}$	At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u} \to \lambda\ln u$; $\alpha, \beta, \delta, \lambda \neq 0$	M1
	$= \frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x\ \{+c\}$	$\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x$, simplified or un-simplified with or without $+c$ **Note:** $\ln(e^x)$ needs to be simplified to x for this mark	A1
			[3]

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8.

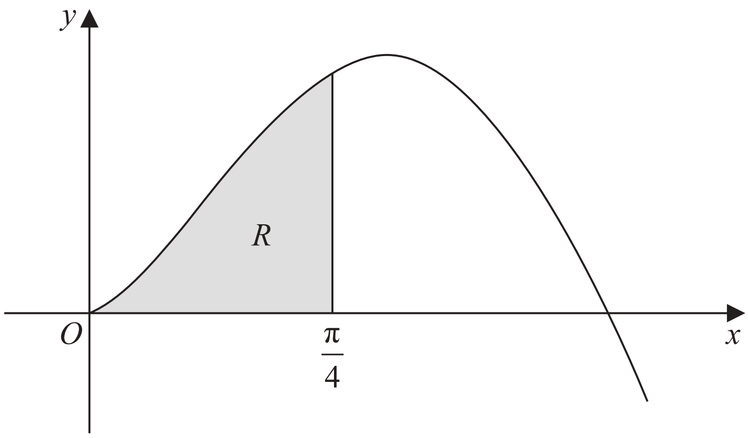

Figure 3

(a) Find $\int x\cos 4x \, \mathrm{d}x$

(3)

Figure 3 shows part of the curve with equation $y = \sqrt{x}\sin 2x, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of this solid of revolution, giving your answer in its simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Question Number	Scheme	Notes	Marks
8. (a)	$\left\{\int x\cos 4x\,\mathrm{d}x\right\}$ $= \frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$	$\pm\alpha x\sin 4x \pm \beta\int \sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$; $\alpha,\ \beta \neq 0$	M1
		$\frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$ **Can be simplified or un-simplified**	A1
	$= \frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\ \{+c\}$	$\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x$ o.e. with or without $+c$ **Can be simplified or un-simplified**	A1
	Note: You can ignore subsequent working following on from a correct solution		**[3]**
(b) **Way 1**	$\{V=\}\ \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$	$\pi\int\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied	B1
	$\left\{\int x\sin^2 2x\,\mathrm{d}x = \right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x = 1-2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral Can be implied.	M1
		Simplifies $\int x\sin^2 2x\,\{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	A1
	$\left\{\int\left(\frac{1}{2}x - \frac{1}{2}x\cos 4x\right)\mathrm{d}x\right\}$ $= \frac{1}{4}x^2 - \frac{1}{2}\left(\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right)\{+c\}$	Integrates to give $\pm Ax^2 \pm Bx\sin 4x \pm C\cos 4x$; $A, B, C \neq 0$ which can be simplified or un-simplified. **Note:** Allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b)	M1
	$\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x = \left[\frac{1}{4}x^2 - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$		
	$= \left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \left(0 - 0 - \frac{1}{32}\cos 0\right)$	**dependent on the previous M mark** see notes	dM1
	$= \left(\frac{\pi^2}{64} + \frac{1}{32}\right) - \left(-\frac{1}{32}\right) = \frac{\pi^2}{64} + \frac{1}{16}$		
	So, $V = \pi\left(\frac{\pi^2}{64} + \frac{1}{16}\right)$ or $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32} + \frac{1}{8}\right)$ o.e.	two term exact answer	A1 o.e.
			[6]
			9
	Question 8 Notes		

	SC	<u>**Special Case for the 2nd M and 3rd M mark for those who use their answer from part (a)**</u> You can apply the 2nd M and 3rd M marks for integration of the form $\pm Ax^2 \pm$ (their answer to part (*a*)) where their answer to part (a) is in the form • $\pm Bx\sin kx \pm C\cos px$ to give $\pm Ax^2 \pm Bx\sin kx \pm C\cos px$ • $\pm Bx\sin kx \pm C\sin px$ to give $\pm Ax^2 \pm Bx\sin kx \pm C\sin px$ • $\pm Bx\cos kx \pm C\sin px$ to give $\pm Ax^2 \pm Bx\cos kx \pm C\sin px$ • $\pm Bx\cos kx \pm C\cos px$ to give $\pm Ax^2 \pm Bx\cos kx \pm C\cos px$ $k, p \neq 0$, k, p can be 1

Question Number	Scheme	Notes	Marks
8. (b) **Way 2**	$\{V=\}\ \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$	$\pi\int\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied	B1
	$\left\{\int x\sin^2 2x\,\mathrm{d}x=\right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x = 1-2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral. Can be implied	M1
		Simplifies $\int x\sin^2 2x\,\{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$ **Note:** This mark can be implied for stating $u=x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1-\cos 4x}{2}$ **or** $u=\frac{1}{2}x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=1-\cos 4x$	A1
	$=x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\int\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)\mathrm{d}x$		
	$=x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\left(\frac{1}{4}x^2+\frac{1}{32}\cos 4x\right)\{+c\}$	Integrates to give $\pm Ax^2 \pm Bx\sin 4x \pm C\cos 4x;\ A, B, C\neq 0$ or an expression that can be simplified to this form	M1 **(B1 on ePEN)**
	$\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x=\left[\frac{1}{4}x^2-\frac{1}{8}x\sin 4x-\frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$		
	$=\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2-\frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right)-\frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right)-\left(0-0-\frac{1}{32}\cos 0\right)$	**dependent on the previous M mark** see notes	dM1
	$=\left(\frac{\pi^2}{64}+\frac{1}{32}\right)-\left(-\frac{1}{32}\right)=\frac{\pi^2}{64}+\frac{1}{16}$		
	So, $V=\pi\left(\frac{\pi^2}{64}+\frac{1}{16}\right)$ or $\frac{1}{64}\pi^3+\frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32}+\frac{1}{8}\right)$ o.e.		A1 o.e.
			[6]

		Question 8 Notes Continued
8. (a)	**SC**	Give ***Special Case*** M1A0A0 for writing down the correct "by parts" formula and using $u=x,\ \frac{\mathrm{d}v}{\mathrm{d}x}=\cos 4x,$ but making only one error in the application of the correct formula
(b)	**Note**	You can imply B1 for seeing $\pi\int y^2\{\mathrm{d}x\}$, followed by $y^2=\left(\sqrt{x}\sin 2x\right)^2$ or $y^2=x\sin^2 2x$
	Note	If the form $\cos 4x=\cos^2 2x-\sin^2 2x$ or $\cos 4x = 2\cos^2 2x-1$ is used, the 1st M cannot be gained until $\cos^2 2x$ has been replaced by $\cos^2 2x = 1-\sin^2 2x$ and the result is applied to their integral
	Note	Mixing x's and e.g. θ's : Condone $\cos 4\theta = 1-2\sin^2 2\theta,\ \sin^2 2\theta=\frac{1-\cos 4\theta}{2}$ or $\lambda\sin^2 2\theta=\lambda\left(\frac{1-\cos 4\theta}{2}\right)$ if recovered in their integration
	Final M1	**Complete** method of applying limits of $\frac{\pi}{4}$ and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bx\sin 4x \pm C\cos 4x;\ A, B, C\neq 0$ and subtracting the correct way round.
	Note	For the final M1 mark in Way 1, allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b)

		Question 8 Notes Continued
8. (b)	**Note**	Evidence of a proper consideration of the limit of 0 on $\cos 4x$ **where applicable** is needed for the final M mark E.g. $\left[\frac{1}{4}x^2 - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}} =$ • $= \left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) + \frac{1}{32}$ is final M1 • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - 0$ is final M0 • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \frac{1}{32}$ is final M0 (adding) • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \left(\frac{1}{32}\right)$ is final M1 (condone) • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - (0+0+0)$ is final M0
8. (b)	**Note**	**Alternative Method**: $\left\{\begin{matrix} u = \sin^2 2x & \frac{dv}{dx} = x \\ \frac{du}{dx} = 2\sin 4x & v = \frac{1}{2}x^2 \end{matrix}\right\}, \left\{\begin{matrix} u = x^2 & \frac{dv}{dx} = \sin 4x \\ \frac{du}{dx} = 2x & v = -\frac{1}{4}\cos 4x \end{matrix}\right\}$ $\int x\sin^2 2x\,dx$ $= \frac{1}{2}x^2\sin^2 2x - \int \frac{1}{2}x^2(2\sin 4x)dx$ $= \frac{1}{2}x^2\sin^2 2x - \int x^2\sin 4x\,dx$ $= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x - \int 2x.\left(-\frac{1}{4}\cos 4x\right)dx\right)$ $= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x + \frac{1}{2}\int x\cos 4x\,dx\right)$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\int x\cos 4x\,dx$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\left(\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right)\ \{+c\}$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\ \{+c\}$ $V = \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2 dx = \pi\left(\frac{\pi^2}{64} + \frac{1}{16}\right)$ or $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32} + \frac{1}{8}\right)$ o.e.

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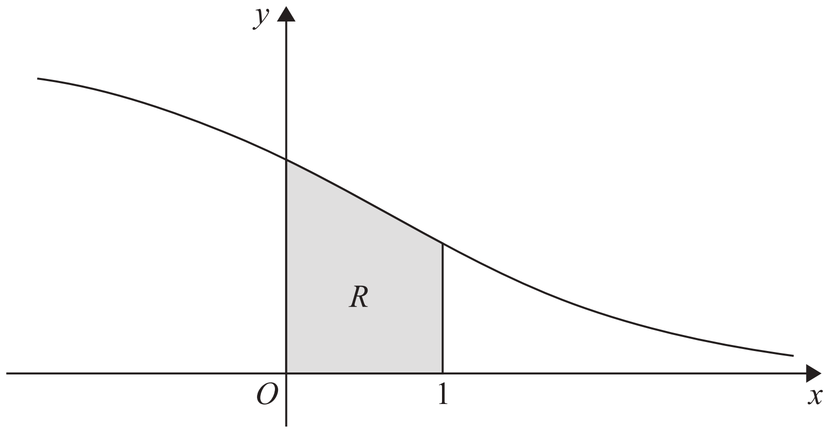

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{6}{(e^x + 2)}$, $x \in \mathbb{R}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 1$

The table below shows corresponding values of x and y for $y = \dfrac{6}{(e^x + 2)}$

x	0	0.2	0.4	0.6	0.8	1
y	2		1.71830	1.56981	1.41994	1.27165

(a) Complete the table above by giving the missing value of y to 5 decimal places.
(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.
(3)

(c) Use the substitution $u = e^x$ to show that the area of R can be given by

$$\int_a^b \frac{6}{u(u+2)}\,du$$

where a and b are constants to be determined.
(2)

(d) Hence use calculus to find the exact area of R.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]
(6)

Question Number	Scheme	Notes	Marks
3.	x: 0, 0.2, 0.4, 0.6, 0.8, 1 ; y: 2, ***1.8625426...***, 1.71830, 1.56981, 1.41994, 1.27165	$y = \dfrac{6}{(2+e^x)}$	
(a)	$\{\text{At } x = 0.2,\}\ y = 1.86254$ (5 dp)	1.86254	B1 **cao**
	Note: Look for this value on the given table or in their working.		**[1]**
(b)	$\frac{1}{2}(0.2)\left[2+1.27165+2\left(\text{their } 1.86254+1.71830+1.56981+1.41994\right)\right]$	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{10}$ or $\frac{1}{2}\times\frac{1}{5}$	B1 o.e.
		For structure of $[.......]$	M1
	$\left\{=\frac{1}{10}(16.41283)\right\}=1.641283=1.6413$ (4 dp)	anything that rounds to 1.6413	A1
			[3]
(c)	$\{u = e^x \text{ or } x = \ln u \Rightarrow\}$		
	$\frac{du}{dx}=e^x$ **or** $\frac{du}{dx}=u$ **or** $\frac{dx}{du}=\frac{1}{u}$ **or** $du = u\,dx$ etc., **and** $\int\frac{6}{(e^x+2)}dx=\int\frac{6}{(u+2)u}du$	**See notes**	B1 *
	$\{x=0\}\Rightarrow a=e^0\Rightarrow \underline{a=1}$ $\{x=1\}\Rightarrow b=e^1\Rightarrow \underline{b=e}$	$a=1$ and $b=e$ or $b=e^1$ or evidence of $0\to1$ and $1\to e$	B1
	NOTE: 1st B1 mark CANNOT be recovered for work in part (d) **NOTE: 2nd B1 mark CAN be recovered for work in part (d)**		**[2]**
(d) **Way 1**	$\frac{6}{u(u+2)}\equiv\frac{A}{u}+\frac{B}{(u+2)}$ $\Rightarrow 6\equiv A(u+2)+Bu$	Writing $\frac{6}{u(u+2)}\equiv\frac{A}{u}+\frac{B}{(u+2)}$, o.e. **or** $\frac{1}{u(u+2)}\equiv\frac{P}{u}+\frac{Q}{(u+2)}$, o.e., and a complete method for finding the value of at least one of **their** A or **their** B (or **their** P or **their** Q)	M1
	$u=0\Rightarrow A=3$ $u=-2\Rightarrow B=-3$	Both **their** $A=3$ **and their** $B=-3$. (Or **their** $P=\frac{1}{2}$ **and their** $Q=-\frac{1}{2}$ with the factor of 6 in front of the integral sign)	A1
	$\int\frac{6}{u(u+2)}du=\int\left(\frac{3}{u}-\frac{3}{(u+2)}\right)du$	Integrates $\frac{M}{u}\pm\frac{N}{u\pm k}$, $M,N,k\neq0$; (i.e. ***a two term partial fraction***) to obtain either $\pm\lambda\ln(\alpha u)$ **or** $\pm\mu\ln(\beta(u\pm k))$; $\lambda,\mu,\alpha,\beta\neq0$	M1
	$=3\ln u-3\ln(u+2)$ or $=3\ln 2u-3\ln(2u+4)$	Integration of both terms is **correctly followed through** from **their** M and from **their** N.	A1 ft
	$\left\{\text{So}\left[3\ln u-3\ln(u+2)\right]_1^e\right\}$ $=\left(3\ln(e)-3\ln(e+2)\right)-\left(3\ln1-3\ln3\right)$ **[Note:** A proper consideration of the limit of $u=1$ is required for this mark]	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b>0, b\neq1, a>0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$=3-3\ln(e+2)+3\ln3$ **or** $3(1-\ln(e+2)+\ln3)$ **or** $3+3\ln\left(\frac{3}{e+2}\right)$ **or** $3\ln\left(\frac{e}{e+2}\right)-3\ln\left(\frac{1}{3}\right)$ **or** $3-3\ln\left(\frac{e+2}{3}\right)$ **or** $3\ln\left(\frac{3e}{e+2}\right)$ **or** $\ln\left(\frac{27e^3}{(e+2)^3}\right)$	**see notes**	A1 **cso**
	Note: Allow e^1 in place of e for the final A1 mark.		**[6]**
	Note: Give final A0 for $3-3\ln e+2+3\ln3$ (i.e. bracketing error) unless recovered.		**12**
	Note: Give final A0 for $3-3\ln(e+2)+3\ln3-3\ln1$, where $3\ln1$ has not been simplified to 0		
	Note: Give final A0 for $3\ln e-3\ln(e+2)+3\ln3$, where $3\ln e$ has not been simplified to 3		

Question 3 Notes

3. (b)	**Note**	**M1:** Do not allow an extra y-value ***or*** a repeated y value in their [...] Do not allow an omission of a y-ordinate in their [...] for M1 **unless** they give the correct answer of awrt 1.6413, in which case both M1 and A1 can be scored.
	Note	**A1:** Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.64150274...)
	Note	Full marks can be gained in part (b) for awrt 1.6413 even if B0 is given in part (a)
	Note	Award B1M1A1 for $\frac{1}{10}(2+1.27165)+\frac{1}{5}(\text{their } 1.86254+1.71830+1.56981+1.41994)=\text{awrt } 1.6413$
		Bracketing mistakes: Unless the final answer implies that the calculation has been done correctly Award B1M0A0 for $\frac{1}{2}(0.2)+2+2(\text{their } 1.86254+1.71830+1.56981+1.41994)+1.27165 \quad (=16.51283)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2+1.27165)+2(\text{their } 1.86254+1.71830+1.56981+1.41994) \quad (=13.468345)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2)+2(\text{their } 1.86254+1.71830+1.56981+1.41994)+1.27165 \quad (=14.61283)$
		Alternative method: Adding individual trapezia $\text{Area} \approx 0.2\times\left[\frac{2+"1.86254"}{2}+\frac{"1.86254"+1.71830}{2}+\frac{1.71830+1.56981}{2}+\frac{1.56981+1.41994}{2}+\frac{1.41994+1.27165}{2}\right]$ $=1.641283$ **B1** 0.2 and a divisor of 2 on all terms inside brackets **M1** First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2 **A1** anything that rounds to 1.6413
3. (c)	**1st B1**	Must start from either • $\int y\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(\mathrm{e}^x+2)}\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u$, with integral sign and $\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ **and** state either $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$ **and** end at $\int \frac{6}{u(u+2)}\,\mathrm{d}u$, with integral sign and $\mathrm{d}u$, **with no incorrect working.**
	Note	So, just writing $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **and** $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x=\int \frac{6}{u(u+2)}\,\mathrm{d}u$ is sufficient for 1st B1
	Note	Give 2nd B0 for $b=2.718...$, without reference to $a=1$ and $b=\mathrm{e}$ or $b=\mathrm{e}^1$
	Note	You can also give the 1st B1 mark for using a reverse process. i.e. Proceeding from $\int \frac{6}{u(u+2)}\,\mathrm{d}u$ to $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x$, **with no incorrect working**, and stating either $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$
3. (d)	**Note**	Give final A0 for $3-3\ln(\mathrm{e}+2)+3\ln 3$ simplifying to $1-\ln(\mathrm{e}+2)+\ln 3$ (i.e. dividing their correct final answer by 3) Otherwise, you can ignore incorrect working (isw) following on from a correct exact value.
	Note	A decimal answer of 1.641502724... (without a correct **exact** answer) is final A0
	Note	$\left[-3\ln(u+2)+3\ln u\right]_1^{\mathrm{e}}$ followed by awrt 1.64 (without a correct **exact** answer) is final M1A0

		Question 3 Notes Continued
3. (d)	**Note**	**BE CAREFUL! Candidates will assign their own "A" and "B" for this question.**
	Note	***Writing down*** $\dfrac{6}{(u+2)u}$ in the form $\dfrac{A}{(u+2)}+\dfrac{B}{u}$ with at least one of A or B correct is 1st M1
	Note	***Writing down*** $\dfrac{6}{(u+2)u}$ as $\dfrac{-3}{(u+2)}+\dfrac{3}{u}$ is 1st M1 1st A1.
	Note	**Condone** $\int\left(\dfrac{3}{u}-\dfrac{3}{(u+2)}\right)\mathrm{d}u$ to give $3\ln u - 3\ln u+2$ (poor bracketing) for 2nd A1.
	Note	**Award M0A0M1A1ft** for a candidate who writes down e.g. $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u = \int\left(\dfrac{6}{u}+\dfrac{6}{(u+2)}\right)\mathrm{d}u = 6\ln u + 6\ln(u+2)$ **AS EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ **AS PARTIAL FRACTIONS.**
	Note	**Award M0A0M0A0** for a candidate who writes down $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u = 6\ln u + 6\ln(u+2)$ **or** $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u = \ln u + 6\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	**Award M1A1M1A1** for a candidate who writes down $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u = 3\ln u - 3\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	If they lose the "6" and find $\int_1^e\dfrac{1}{u(u+2)}\mathrm{d}u$ we can allow a maximum of M1A0M1A1ftM1A0

	Question 3 Notes Continued	
3. (d) **Way 2**	$\left\{\int \frac{6}{u^2+2u}\,du = \int \frac{3(2u+2)}{u^2+2u}\,du - \int \frac{6u}{u^2+2u}\,du\right\}$	
	$= \int \frac{3(2u+2)}{u^2+2u}\,du - \int \frac{6}{u+2}\,du$ — $\int \frac{\pm\alpha(2u+2)}{u^2+2u}\{du\} \pm \int \frac{\delta}{u+2}\{du\}$, $\alpha, \beta, \delta \neq 0$	M1
	Correct expression	A1
	$= 3\ln(u^2+2u) - 6\ln(u+2)$ — Integrates $\frac{\pm M(2u+2)}{u^2+2u} \pm \frac{N}{u \pm k}$, $M, N, k \neq 0$, to obtain any one of $\pm\lambda\ln(u^2+2u)$ **or** $\pm\mu\ln(\beta(u \pm k))$; $\lambda, \mu, \beta \neq 0$	M1
	Integration of both terms is **correctly followed through** from **their** M and from **their** N	A1 ft
	$\left\{\text{So, } \left[3\ln(u^2+2u) - 6\ln(u+2)\right]_1^{e}\right\}$ $= \left(3\ln(e^2+2e) - 6\ln(e+2)\right) - \left(3\ln 3 - 6\ln 3\right)$ — **dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b > 0, b \neq 1, a > 0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$= 3\ln(e^2+2e) - 6\ln(e+2) + 3\ln 3$ — $3\ln(e^2+2e) - 6\ln(e+2) + 3\ln 3$	A1 o.e.
		[6]
3. (d) **Way 3**	Applying $u = \theta - 1$	
	$\left\{\int_1^{e} \frac{6}{u(u+2)}\,du = \right\} \int_2^{1+e} \frac{6}{(\theta-1)(\theta+1)}\,d\theta = \int_2^{1+e} \frac{6}{\theta^2-1}\,du = \left[3\ln\left(\frac{\theta-1}{\theta+1}\right)\right]_2^{1+e}$	M1A1M1A1
	$= 3\ln\left(\frac{1+e-1}{e+1+1}\right) - 3\ln\left(\frac{2-1}{2+1}\right) = 3\ln\left(\frac{e}{e+2}\right) - 3\ln\left(\frac{1}{3}\right)$ — 3rd M mark is dependent on 2nd M mark	dM1A1
		[6]

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5.

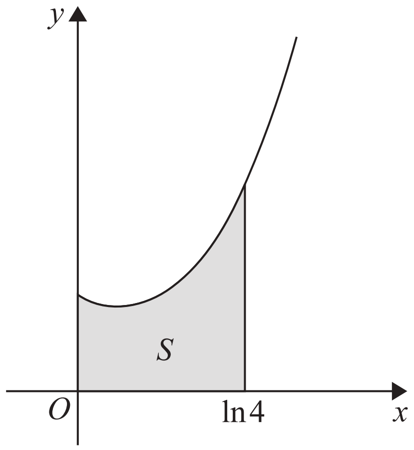

Diagram not drawn to scale

Figure 2

The finite region S, shown shaded in Figure 2, is bounded by the y-axis, the x-axis, the line with equation $x = \ln 4$ and the curve with equation

$$y = e^x + 2e^{-x}, \quad x \geqslant 0$$

The region S is rotated through 2π radians about the x-axis.

Use integration to find the exact value of the volume of the solid generated. Give your answer in its simplest form.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(7)

Question Number	Scheme	Notes	Marks
5. **Way 1**	$y = \mathrm{e}^x + 2\mathrm{e}^{-x},\ x \geq 0$		
	$\{V =\}\ \pi \int_0^{\ln 4} \left(\mathrm{e}^x + 2\mathrm{e}^{-x}\right)^2 \mathrm{d}x$	For $\pi \int \left(\mathrm{e}^x + 2\mathrm{e}^{-x}\right)^2$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\} \int_0^{\ln 4} \left(\mathrm{e}^{2x} + 4\mathrm{e}^{-2x} + 4\right)\mathrm{d}x$	Expands $\left(\mathrm{e}^x + 2\mathrm{e}^{-x}\right)^2 \to \pm\alpha\mathrm{e}^{2x} \pm \beta\mathrm{e}^{-2x} \pm \delta$ where $\alpha, \beta, \delta \neq 0$. Ignore π, integral sign, limits and $\mathrm{d}x$. This can be implied by later work.	M1
	$= \{\pi\}\left[\frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^{-2x} + 4x\right]_0^{\ln 4}$	Integrates at least one of either $\pm\alpha\mathrm{e}^{2x}$ to give $\pm\frac{\alpha}{2}\mathrm{e}^{2x}$ or $\pm\beta\mathrm{e}^{-2x}$ to give $\pm\frac{\beta}{2}\mathrm{e}^{-2x}$ $\alpha, \beta \neq 0$	M1
		dependent on the 2nd M mark $\mathrm{e}^{2x} + 4\mathrm{e}^{-2x} \to \frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^{-2x}$, which can be simplified or un-simplified	A1
		$4 \to 4x$ or $4\mathrm{e}^0 x$	B1 cao
	$= \{\pi\}\left(\left(\frac{1}{2}\mathrm{e}^{2(\ln 4)} - 2\mathrm{e}^{-2(\ln 4)} + 4(\ln 4)\right) - \left(\frac{1}{2}\mathrm{e}^0 - 2\mathrm{e}^0 + 4(0)\right)\right)$	**dependent on the previous method mark.** Some evidence of applying limits of $\ln 4$ o.e. and 0 to a changed function in x and subtracts the correct way round. **Note:** A proper consideration of the limit of 0 is required.	dM1
	$= \{\pi\}\left(\left(8 - \frac{1}{8} + 4\ln 4\right) - \left(\frac{1}{2} - 2\right)\right)$		
	$= \frac{75}{8}\pi + 4\pi\ln 4$ **or** $\frac{75}{8}\pi + 8\pi\ln 2$ **or** $\pi\left(\frac{75}{8} + 4\ln 4\right)$ **or** $\pi\left(\frac{75}{8} + 8\ln 2\right)$ **or** $\frac{75}{8}\pi + \ln 2^{8\pi}$ **or** $\frac{75}{8}\pi + \pi\ln 256$ **or** $\ln\left(2^{8\pi}\mathrm{e}^{\frac{75}{8}\pi}\right)$ **or** $\frac{1}{8}\pi\left(75 + 32\ln 4\right)$, etc		A1 isw
			[7]
			7

		Question 5 Notes
5.	**Note**	π is only required for the 1st B1 mark and the final A1 mark.
	Note	Give 1st B0 for writing $\pi\int y^2\,\mathrm{d}x$ followed by $2\pi\int\left(\mathrm{e}^x + 2\mathrm{e}^{-x}\right)^2\mathrm{d}x$
	Note	Give 1st M1 for $\left(\mathrm{e}^x + 2\mathrm{e}^{-x}\right)^2 \to \mathrm{e}^{2x} + 4\mathrm{e}^{-2x} + 2\mathrm{e}^0 + 2\mathrm{e}^0$ because $\delta = 2\mathrm{e}^0 + 2\mathrm{e}^0$
	Note	A decimal answer of 46.8731... or $\pi(14.9201...)$ (without a correct **exact** answer) is A0
	Note	$\pi\left[\frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^{-2x} + 4x\right]_0^{\ln 4}$ followed by awrt 46.9 (without a correct **exact** answer) is final dM1A0
	Note	Allow exact equivalents which should be in the form $a\pi + b\pi\ln c$ or $\pi(a + b\ln c)$, where $a = \frac{75}{8}$ or $9\frac{3}{8}$ or 9.375. Do not allow $a = \frac{150}{16}$ or $9\frac{6}{16}$
	Note	Give B1M0M1A1B0M1A0 for the common response $\pi\int_0^{\ln 4}\left(\mathrm{e}^x + 2\mathrm{e}^{-x}\right)^2\mathrm{d}x \to \pi\int_0^{\ln 4}\left(\mathrm{e}^{2x} + 4\mathrm{e}^{-2x}\right)\mathrm{d}x = \pi\left[\frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^{-2x}\right]_0^{\ln 4} = \frac{75}{8}\pi$

Question Number	Scheme	Notes	Marks
5. **Way 2**	$y = e^x + 2e^{-x},\ x \geq 0$		
	$\{V =\}\ \pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx$	For $\pi\int\left(e^x + 2e^{-x}\right)^2$ Ignore limits and dx. Can be implied.	B1
	$u = e^x \Rightarrow \frac{du}{dx} = e^x = u$ and $x = \ln 4 \Rightarrow u = 4,\ x = 0 \Rightarrow u = e^0 = 1$		
	$V = \{\pi\}\int_1^4\left(u + \frac{2}{u}\right)^2\frac{1}{u}du = \{\pi\}\int_1^4\left(u^2 + \frac{4}{u^2} + 4\right)\frac{1}{u}du$		
	$= \{\pi\}\int_1^4\left(u + \frac{4}{u^3} + \frac{4}{u}\right)du$	$\left(e^x + 2e^{-x}\right)^2 \to \pm\alpha u \pm \beta u^{-3} \pm \delta u^{-1}$ where $u = e^x,\ \alpha,\ \beta,\ \delta \neq 0$. Ignore π, integral sign, limits and du. This can be implied by later work.	M1
	$= \{\pi\}\left[\frac{1}{2}u^2 - \frac{2}{u^2} + 4\ln u\right]_1^4$	Integrates at least one of either $\pm\alpha u$ to give $\pm\frac{\alpha}{2}u^2$ or $\pm\beta u^{-3}$ to give $\pm\frac{\beta}{2}u^{-2}$ $\alpha,\ \beta \neq 0$, where $u = e^x$	M1
		dependent on the 2nd M mark $u + 4u^{-3} \to \frac{1}{2}u^2 - 2u^{-2}$, simplified or un-simplified, where $u = e^x$	A1
		$4u^{-1} \to 4\ln u$, where $u = e^x$	B1 cao
	$= \{\pi\}\left(\left(\frac{1}{2}(4)^2 - \frac{2}{(4)^2} + 4\ln 4\right) - \left(\frac{1}{2}(1)^2 - \frac{2}{(1)^2} + 4\ln 1\right)\right)$	**dependent on the previous method mark.** Some evidence of applying limits of 4 and 1 to a changed function in u [or $\ln 4$ o.e. and 0 to an integrated function in x] and subtracts the correct way round.	dM1
	$= \{\pi\}\left(\left(8 - \frac{1}{8} + 4\ln 4\right) - \left(\frac{1}{2} - 2\right)\right)$		
	$= \frac{75}{8}\pi + 4\pi\ln 4$ **or** $\frac{75}{8}\pi + 8\pi\ln 2$ **or** $\pi\left(\frac{75}{8} + 4\ln 4\right)$ **or** $\pi\left(\frac{75}{8} + 8\ln 2\right)$ **or** $\frac{75}{8}\pi + \ln 2^{8\pi}$ **or** $\frac{75}{8}\pi + \pi\ln 256$ **or** $\ln\left(2^{8\pi}e^{\frac{75}{8}\pi}\right)$ **or** $\frac{1}{8}\pi\left(75 + 32\ln 4\right)$, etc		A1 isw
			[7]

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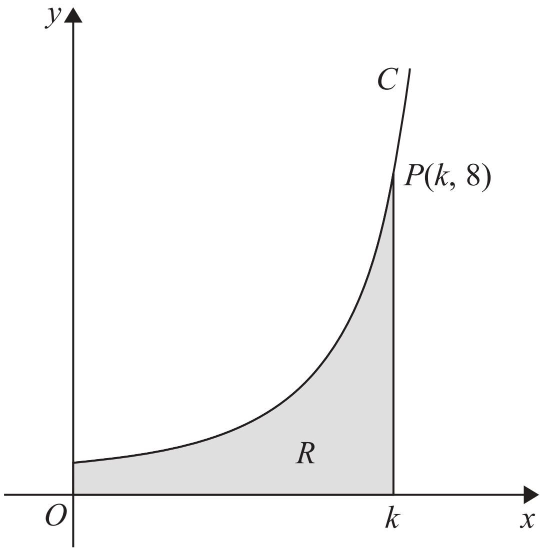

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta\sin\theta, \qquad y = \sec^3\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda\int_{\alpha}^{\beta}\left(\theta\sec^2\theta + \tan\theta\sec^2\theta\right)\mathrm{d}\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

Question Number	Scheme	Notes	Marks
8.	$x = 3\theta\sin\theta,\ \ y = \sec^3\theta,\ \ 0 \le \theta < \frac{\pi}{2}$		
(a)	$\{\text{When } y=8,\}\ 8=\sec^3\theta \Rightarrow \cos^3\theta = \frac{1}{8} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $k \text{ (or } x) = 3\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$	Sets $y=8$ to find θ **and** attempts to substitute their θ into $x = 3\theta\sin\theta$	M1
	so $k \text{ (or } x) = \frac{\sqrt{3}\pi}{2}$	$\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	A1
	Note: Obtaining two value for k without accepting the correct value is final A0		**[2]**
(b)	$\frac{dx}{d\theta} = 3\sin\theta + 3\theta\cos\theta$	$3\theta\sin\theta \to 3\sin\theta + 3\theta\cos\theta$ Can be implied by later working	B1
	$\left\{\int y\frac{dx}{d\theta}\{d\theta\}\right\} = \int(\sec^3\theta)(3\sin\theta + 3\theta\cos\theta)\{d\theta\}$	Applies $(\pm K\sec^3\theta)\left(\text{their } \frac{dx}{d\theta}\right)$ Ignore integral sign and $d\theta$; $K \neq 0$	M1
	$= 3\int \theta\sec^2\theta + \tan\theta\sec^2\theta\, d\theta$	Achieves the correct result no errors in their working, e.g. bracketing or manipulation errors. **Must have** integral sign and $d\theta$ in their final answer.	A1 *
	$x=0$ and $x=k \Rightarrow \underline{\alpha = 0}$ and $\underline{\beta = \frac{\pi}{3}}$	$\alpha = 0$ and $\beta = \frac{\pi}{3}$ or evidence of $0 \to 0$ and $k \to \frac{\pi}{3}$	B1
	Note: The work for the final B1 mark must be seen in part (b) only.		**[4]**
(c) **Way 1**	$\left\{\int \theta\sec^2\theta\, d\theta\right\} = \theta\tan\theta - \int \tan\theta\{d\theta\}$	$\theta\sec^2\theta \to A\theta g(\theta) - B\int g(\theta),\ A>0,\ B>0,$ where $g(\theta)$ is a trigonometric function in θ **and** $g(\theta) =$ their $\int \sec^2\theta\, d\theta$. [Note: $g(\theta) \neq \sec^2\theta$]	M1
		dependent on the previous M mark **Either** $\lambda\theta\sec^2\theta \to A\theta\tan\theta - B\int\tan\theta,\ A>0,\ B>0$ **or** $\theta\sec^2\theta \to \theta\tan\theta - \int\tan\theta$	dM1
	$= \theta\tan\theta - \ln(\sec\theta)$ **or** $= \theta\tan\theta + \ln(\cos\theta)$	$\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec\theta)$ or $\theta\tan\theta + \ln(\cos\theta)$ **or** $\lambda\theta\sec^2\theta \to \lambda\theta\tan\theta - \lambda\ln(\sec\theta)$ or $\lambda\theta\tan\theta + \lambda\ln(\cos\theta)$	A1
	Note: Condone $\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec x)$ or $\theta\tan\theta + \ln(\cos x)$ for A1		
	$\left\{\int \tan\theta\sec^2\theta\, d\theta\right\}$	$\tan\theta\sec^2\theta$ or $\lambda\tan\theta\sec^2\theta \to \pm C\tan^2\theta$ or $\pm C\sec^2\theta$ or $\pm Cu^{-2}$, where $u = \cos\theta$	M1
	$= \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2u^2}$ where $u = \cos\theta$ or $\frac{1}{2}u^2$ where $u = \tan\theta$	$\tan\theta\sec^2\theta \to \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2\cos^2\theta}$ or $\tan^2\theta - \frac{1}{2}\sec^2\theta$ or $0.5u^{-2}$, where $u = \cos\theta$ or $0.5u^2$, where $u = \tan\theta$ or $\lambda\tan\theta\sec^2\theta \to \frac{\lambda}{2}\tan^2\theta$ or $\frac{\lambda}{2}\sec^2\theta$ or $\frac{\lambda}{2\cos^2\theta}$ or $0.5\lambda u^{-2}$, where $u = \cos\theta$ or $0.5\lambda u^2$, where $u = \tan\theta$	A1
	$\{\text{Area}(R)\} = \left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\tan^2\theta\right]_0^{\frac{\pi}{3}}$ **or** $\left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\sec^2\theta\right]_0^{\frac{\pi}{3}}$		
	$= \left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(3)\right) - (0)$ **or** $\left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(4)\right) - \left(\frac{3}{2}\right)$		
	$= \frac{9}{2} + \sqrt{3}\pi - 3\ln 2$ **or** $\frac{9}{2} + \sqrt{3}\pi + 3\ln\left(\frac{1}{2}\right)$ **or** $\frac{9}{2} + \sqrt{3}\pi - \ln 8$ **or** $\ln\left(\frac{1}{8}e^{\frac{9}{2}+\sqrt{3}\pi}\right)$		A1 o.e.
			[6]
			12

Question Number	Scheme	Notes	Marks
8. (c) **Way 2**	**Way 2 for the first 5 marks:** Applying integration by parts on $\int (\theta + \tan\theta)\sec^2\theta\,\mathrm{d}\theta$		
	$\int (\theta\sec^2\theta + \tan\theta\sec^2\theta)\,\mathrm{d}\theta = \int (\theta + \tan\theta)\sec^2\theta\,\mathrm{d}\theta,$	$\left\{ u = \theta + \tan\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta} = 1 + \sec^2\theta;\ \frac{\mathrm{d}v}{\mathrm{d}\theta} = \sec^2\theta \Rightarrow v = \tan\theta = \mathrm{g}(\theta) \right\}$	
	$\mathrm{h}(\theta)$ and $\mathrm{g}(\theta)$ are trigonometric functions in θ **and** $\mathrm{g}(\theta) =$ their $\int \sec^2\theta\,\mathrm{d}\theta$. [**Note:** $\mathrm{g}(\theta) \neq \sec^2\theta$]		
	$= (\theta + \tan\theta)\tan\theta - \int (1 + \sec^2\theta)\tan\theta\,\{\mathrm{d}\theta\}$	$A(\theta + \tan\theta)\mathrm{g}(\theta) - B\int (1 + \mathrm{h}(\theta))\mathrm{g}(\theta),\ A > 0, B > 0$	M1
		dependent on the previous M mark **Either** $\int \left[(\theta + \tan\theta)\sec^2\theta\right] \to A(\theta + \tan\theta)\tan\theta - B\int (1 + \mathrm{h}(\theta))\tan\theta,\ A \neq 0, B > 0$ **or** $(\theta + \tan\theta)\tan\theta - \int (1 + \mathrm{h}(\theta))\tan\theta$	dM1
	$= (\theta + \tan\theta)\tan\theta - \int (\tan\theta + \tan\theta\sec^2\theta)\,\{\mathrm{d}\theta\}$		
	$= (\theta + \tan\theta)\tan\theta - \ln(\sec\theta) - \int \tan\theta\sec^2\theta\,\{\mathrm{d}\theta\}$	$(\theta + \tan\theta)\tan\theta - \ln(\sec\theta)$ o.e. or $\int \left[(\theta + \tan\theta)\tan\theta - \ln(\sec\theta)\right]$ o.e.	A1
	$= (\theta + \tan\theta)\tan\theta - \ln(\sec\theta) - \frac{1}{2}\tan^2\theta$ or $= (\theta + \tan\theta)\tan\theta - \ln(\sec\theta) - \frac{1}{2}\sec^2\theta$ etc.	$\tan\theta\sec^2\theta \to \pm C\tan^2\theta$ or $\pm C\sec^2\theta$	M1
		$(\theta + \tan\theta)\tan\theta - \frac{1}{2}\tan^2\theta$ or $(\theta + \tan\theta)\tan\theta - \frac{1}{2}\sec^2\theta$	A1
	Note	Allow the first two marks in part (c) for $\theta\tan\theta - \int \tan\theta$ embedded in their working	
	Note	Allow the first three marks in part (c) for $\theta\tan\theta - \ln(\sec\theta)$ embedded in their working	
	Note	Allow 3rd M1 2nd A1 marks for either $\tan^2\theta - \frac{1}{2}\tan^2\theta$ or $\tan^2\theta - \frac{1}{2}\sec^2\theta$ embedded in their working	
	Question 8 Notes		
8. (a)	**Note**	Allow M1 for an answer of $k =$ awrt 2.72 without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	Allow M1 for an answer of $k = 3\left(\arccos(\tfrac{1}{2})\right)\sin\left(\arccos(\tfrac{1}{2})\right)$ without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	E.g. allow M1 for $\theta = 60°$, leading to $k = 3(60)\sin(60)$ or $k = 90\sqrt{3}$	

Question 8 Notes Continued

8. (b)	**Note**	To gain A1, $\mathrm{d}\theta$ does not need to appear until they obtain $3\int(\theta\sec^2\theta + \tan\theta\sec^2\theta)\,\mathrm{d}\theta$
	Note	For M1, their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$, where their $\frac{\mathrm{d}x}{\mathrm{d}\theta} \neq 3\theta\sin\theta$, needs to be a trigonometric function in θ
	Note	Writing $\int(\sec^3\theta)(3\sin\theta + 3\theta\cos\theta) = 3\int(\theta\sec^2\theta + \tan\theta\sec^2\theta)\,\mathrm{d}\theta$ is sufficient for B1M1A1
	Note	Writing $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 3\sin\theta + 3\theta\cos\theta$ followed by writing $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta = 3\int(\theta\sec^2\theta + \tan\theta\sec^2\theta)\,\mathrm{d}\theta$ is sufficient for B1M1A1
	Note	The final A mark would be lost for $\int\frac{1}{\cos^3\theta}3\sin\theta + 3\theta\cos\theta = 3\int(\theta\sec^2\theta + \tan\theta\sec^2\theta)\,\mathrm{d}\theta$ [lack of brackets in this particular case].
	Note	Give 2nd B0 for $\alpha = 0$ and $\beta = 60°$, without reference to $\beta = \frac{\pi}{3}$
(c)	**Note**	A decimal answer of 7.861956551... (without a correct **exact** answer) is A0.
	Note	First three marks are for integrating $\theta\sec^2\theta$ with respect to θ
	Note	Fourth and fifth marks are for integrating $\tan\theta\sec^2\theta$ with respect to θ
	Note	Candidates are not penalised for writing $\ln\lvert\sec\theta\rvert$ as either $\ln(\sec\theta)$ or $\ln\sec\theta$
	Note	$\theta\sec^2\theta \to \theta\tan\theta + \ln(\sec\theta)$ **WITH NO INTERMEDIATE WORKING** is M0M0A0
	Note	$\theta\sec^2\theta \to \theta\tan\theta - \ln(\cos\theta)$ **WITH NO INTERMEDIATE WORKING** is M0M0A0
	Note	$\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec\theta)$ **WITH NO INTERMEDIATE WORKING** is M1M1A1
	Note	$\theta\sec^2\theta \to \theta\tan\theta + \ln(\cos\theta)$ **WITH NO INTERMEDIATE WORKING** is M1M1A1
	Note	Writing a correct $uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}$ with $u = \theta,\ \frac{\mathrm{d}v}{\mathrm{d}\theta} = \tan\theta,\ \frac{\mathrm{d}u}{\mathrm{d}\theta} = 1$ and $v =$ their $g(\theta)$ and making one error in the direct application of this formula is 1st M1 only.

8. (c)	Alternative method for finding $\int\tan\theta\sec^2\theta\,\mathrm{d}\theta$		
	$\left\{ u = \tan\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta} = \sec^2\theta;\ \frac{\mathrm{d}v}{\mathrm{d}\theta} = \sec^2\theta \Rightarrow v = \tan\theta \right\}$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \tan^2\theta - \int\tan\theta\sec^2\theta\,\mathrm{d}\theta$ $\Rightarrow 2\int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \tan^2\theta$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \frac{1}{2}\tan^2\theta$	$\tan\theta\sec^2\theta$ or $\to \pm C\tan^2\theta$	M1
		$\tan\theta\sec^2\theta \to \frac{1}{2}\tan^2\theta$	A1
	or $\left\{ u = \sec\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta} = \sec\theta\tan\theta;\ \frac{\mathrm{d}v}{\mathrm{d}\theta} = \sec\theta\tan\theta \Rightarrow v = \sec\theta \right\}$		
	$\Rightarrow \int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \sec^2\theta - \int\sec^2\theta\tan\theta\,\mathrm{d}\theta$ $\Rightarrow 2\int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \sec^2\theta$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta = \frac{1}{2}\sec^2\theta$	$\tan\theta\sec^2\theta$ or $\to \pm C\sec^2\theta$	M1
		$\tan\theta\sec^2\theta \to \frac{1}{2}\sec^2\theta$	A1